Counter Detection

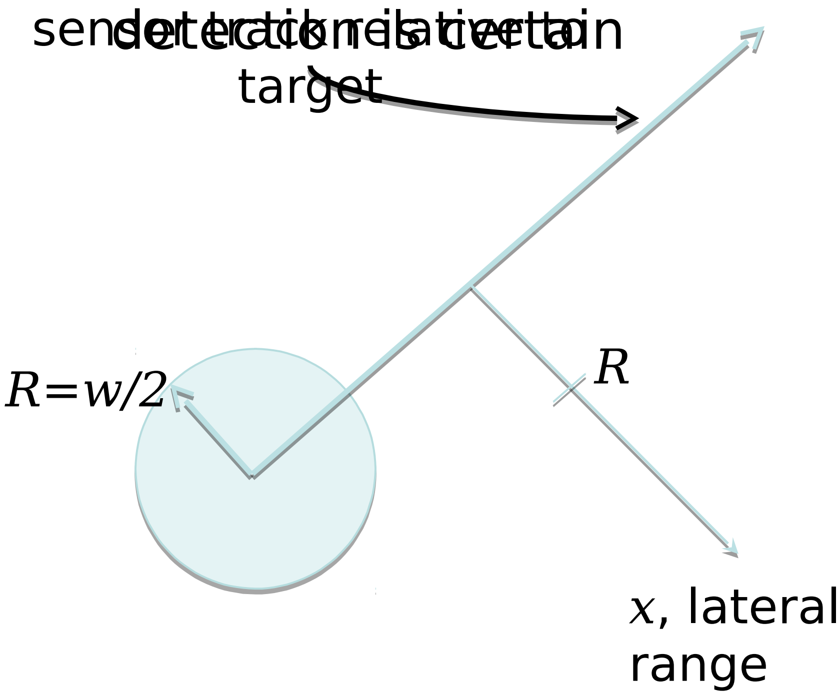
Objectives

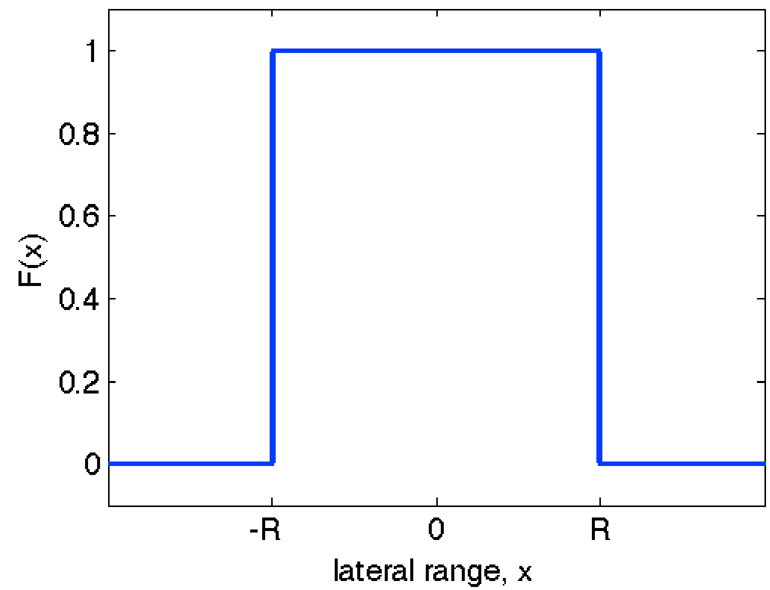
- Compute the effective sweepwidth if the target is trying to evade the searcher

- Compute the effective sweepwidth if the target is trying to approach the searcher

Review

- Recall the definition of the finite range sensor
  - If the target's CPA is within $-R \le x \le R$, target detection is certain





Counter Detection

- Searcher characteristics
  - velocity: v
  - cookie-cutter sensor range: R_s

- Target characteristics
  - velocity: u
  - cookie-cutter sensor range: R_t

- Assumptions
  - $v > u$: searcher faster
  - $R_t > R_s$: target detects searcher first and takes an evasive course in an attempt to avoid detection

Questions

- Under what conditions can the target always evade detection?

- What is the target's effective sweepwidth?
  - w_e
  - Will be less than $2R_s$ because target can evade

Bottom Line

- Evasion scenario: target wants to avoid detection
  - We will primarily focus on this situation

$$w_e = 2R_t \sin\left(\cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right)$$

- Approach scenario: target wants to be detected
  - Rescue situation
  - Target is actually shooter moving in to attack
  - In this case, effective sweep width ***greater*** than $2R_s$

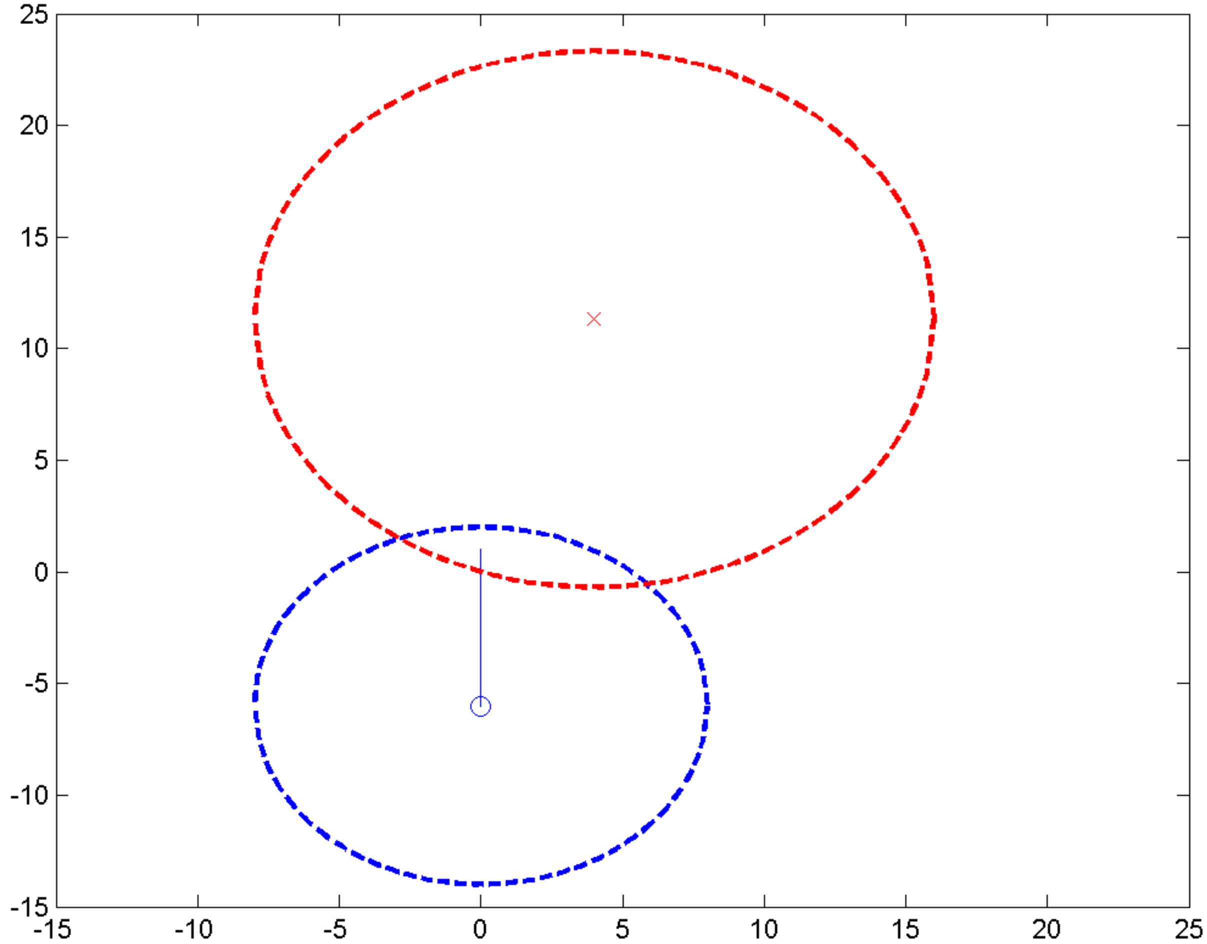
$$w_e = 2R_t \sin\left(\pi - \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right)$$

- Derivation: more tedious geometry/trigonometry

Illustration

- Assume target stationary (relative motion) until counter-detection occurs
- Searcher heads due north

No detections
No counter-detections



Illustration

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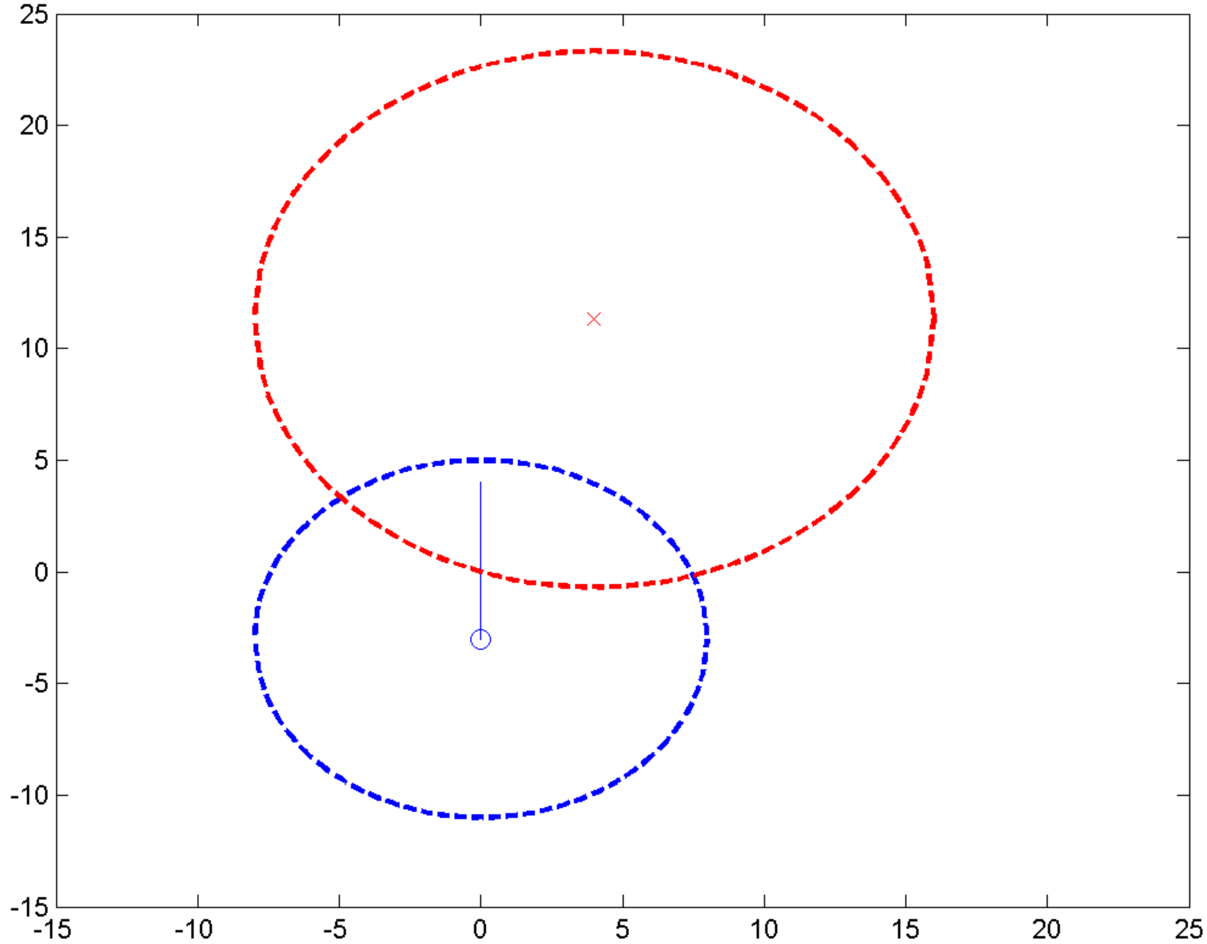
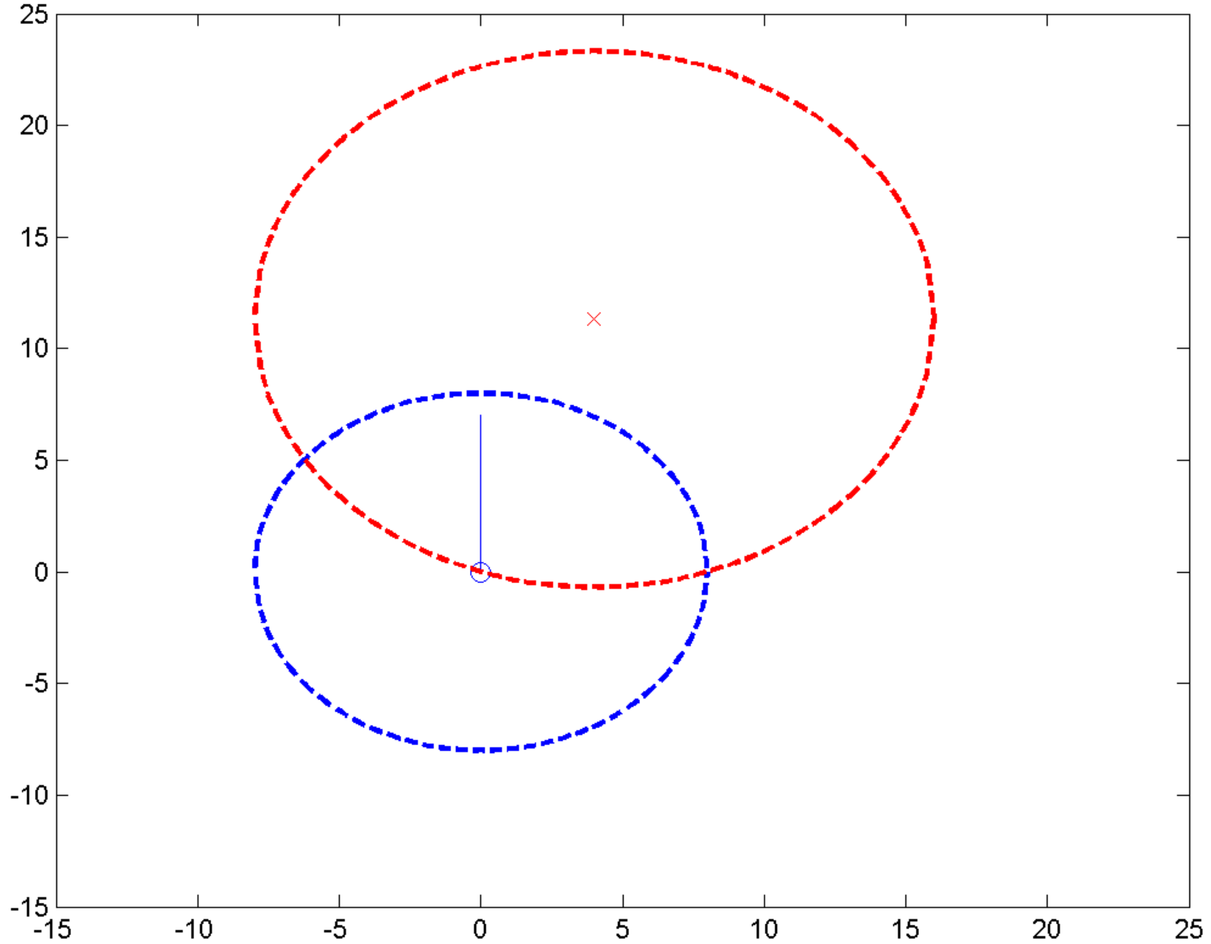


Illustration

- Assume target stationary (relative motion) until counter-detection occurs
- Searcher heads due north

No detections
Red counter-detect

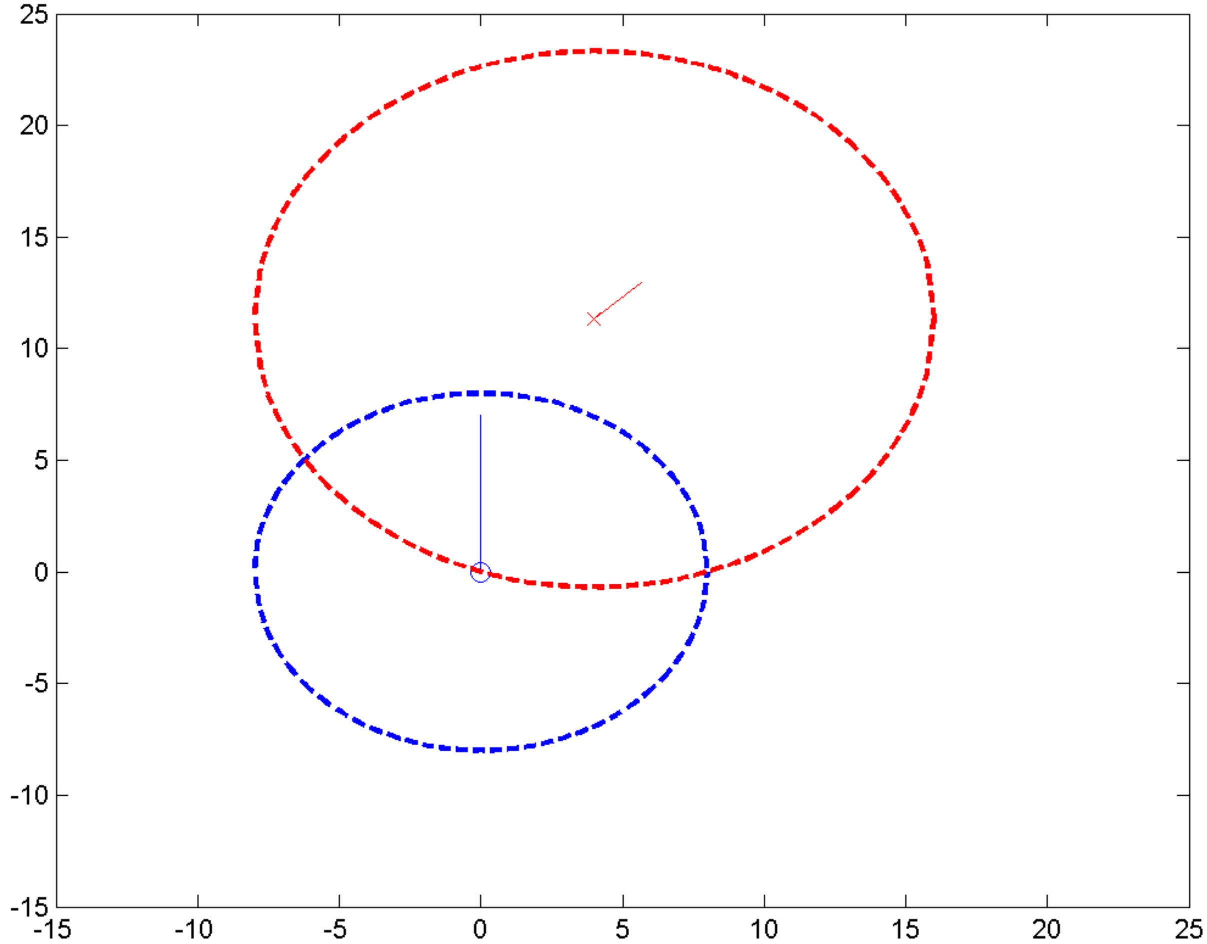
If Red remains stationary, then Blue will detect Red. However, if Red takes evasive actions (intuitively head to the northeast), Red may be able to avoid detection



Illustration

- Assume target stationary (relative motion) until counter-detection occurs
- Searcher heads due north

No detections
Red counter-detect



If Red remains stationary, than Blue will detect Red. However, if Red takes evasive actions (intuitively head to the northeast), Red may be able to avoid detection

Effective Sweepwidth

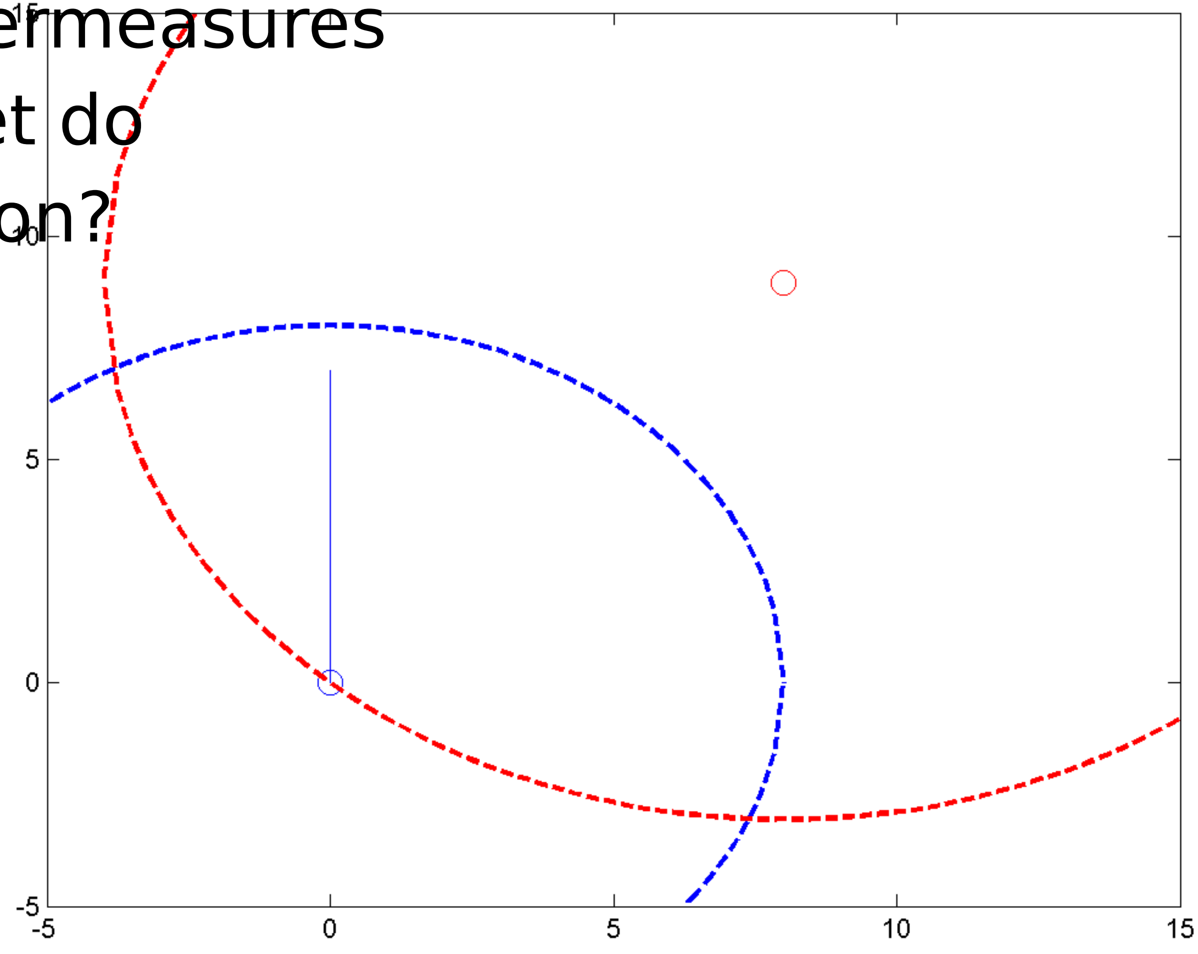
- With counter-detection the target can increase its lateral range
  - The distance at the closest point of approach (CPA)
  - Move away from approaching searcher

- Thus there will be situations where a target without counter-measures will be detected . . .
  - . . . But a target with counter-measures can avoid detection

- Impact: reduction in effective sweepwidth
  - Less than $2*R_s$

Effective Sweepwidth Illustration

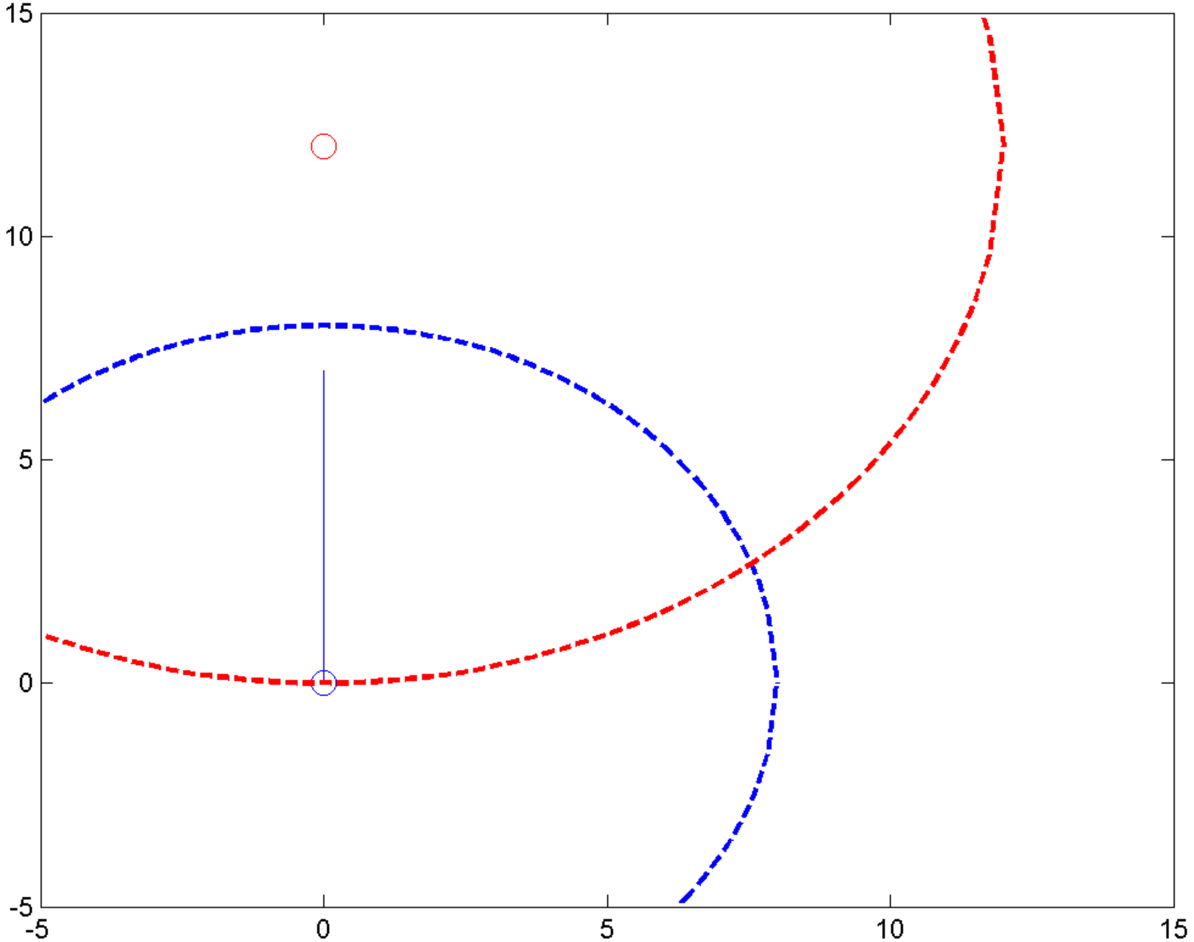
- At time of counter-detection, the target sits at a lateral range of R_s
- Searcher will detect target if the target does not take countermeasures
- What can target do to avoid detection?
  - Head due east



Effective Sweepwidth Illustration

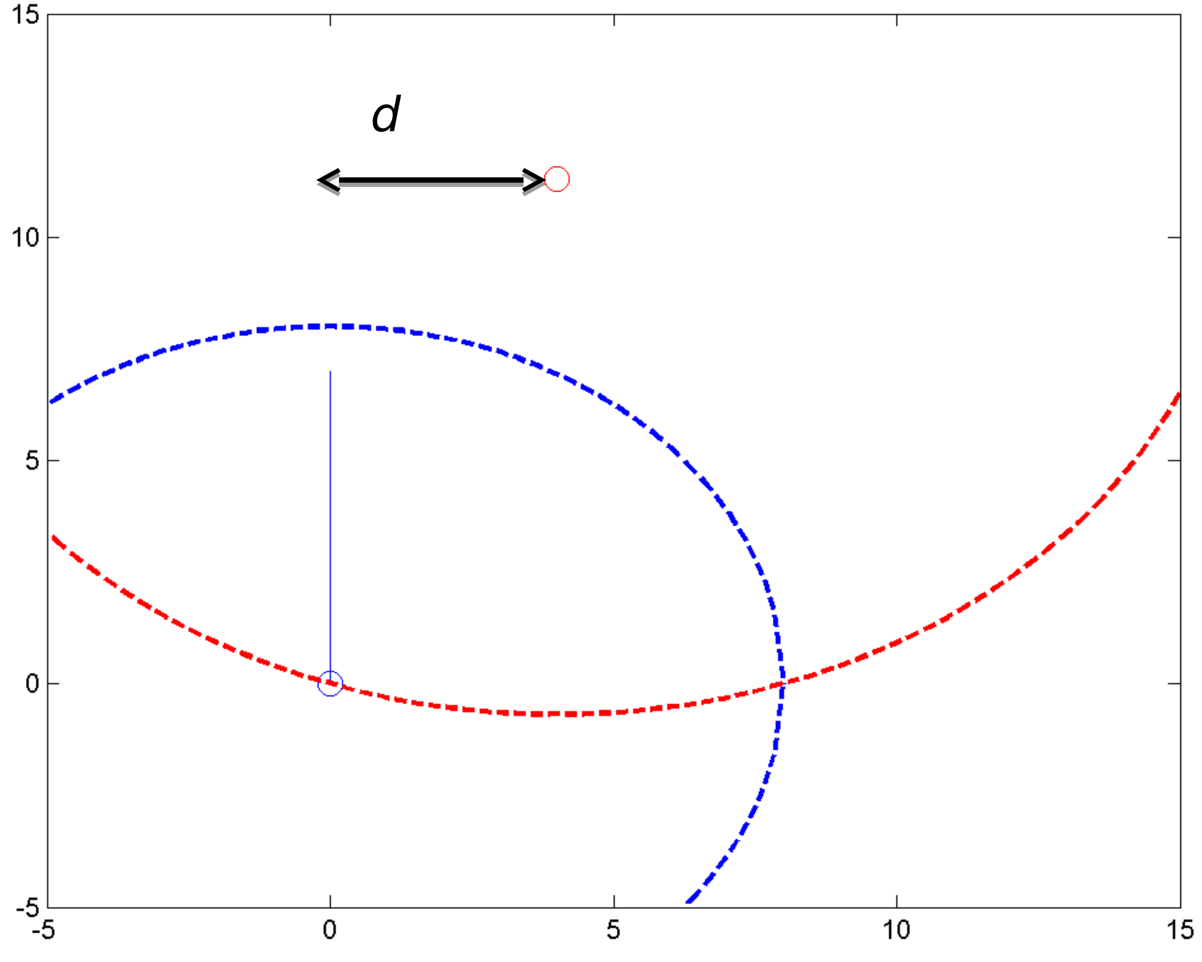
- At time of counter-detection, the target sits at a lateral range of x=0
- Can target avoid detection?

- Possibly, depending on the values of u,v, R_s, and R_t
- Implication:
  - Effective sweepwidth may be 0: searcher can never detect target under any circumstances

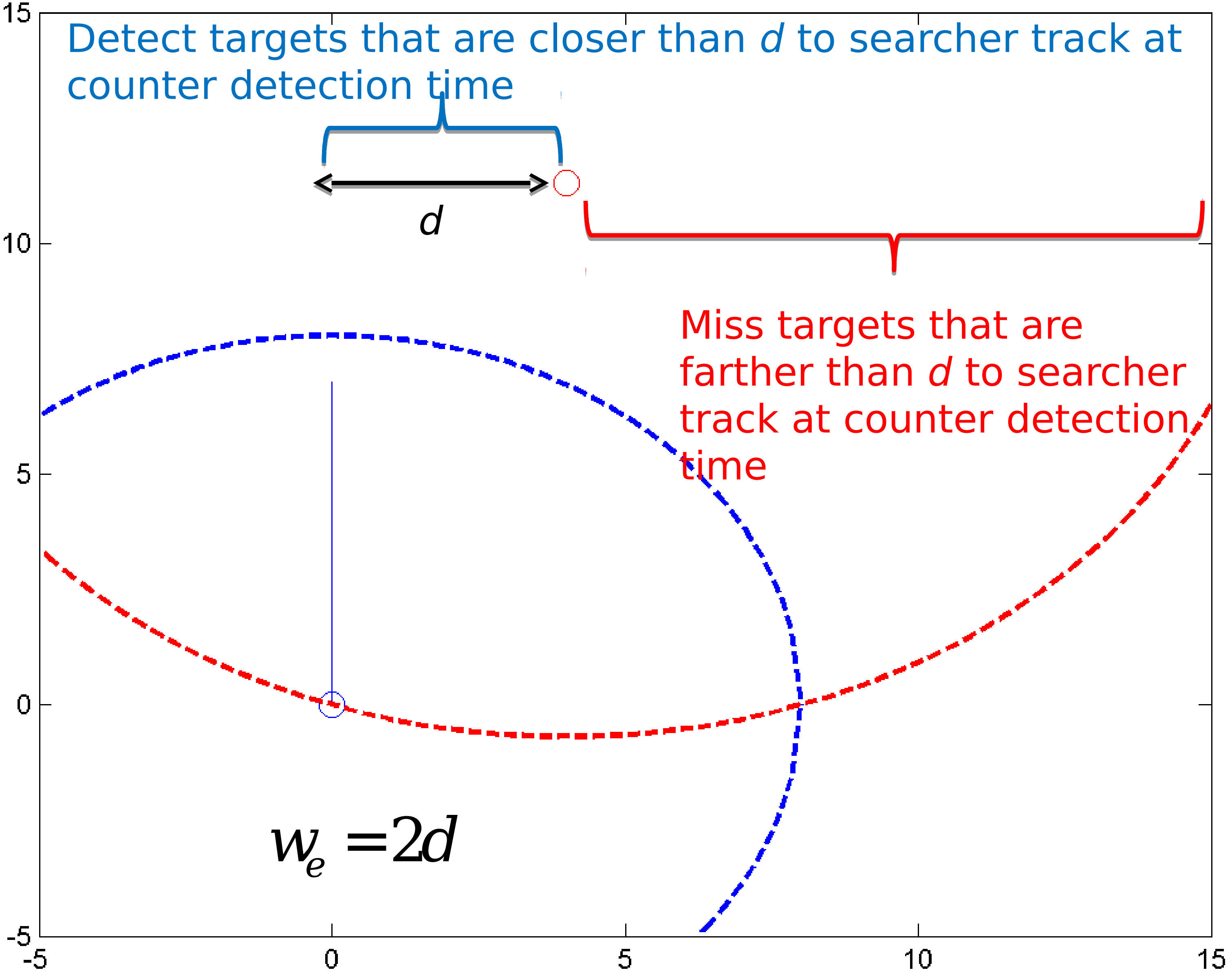


Effective Sweepwidth Illustration

- Want to find a distance *d*, such that searcher will still detect targets with lateral ranges less than *d* at time of counter-detection,
- Effective sweepwidth $w_e = 2d$



Effective Sweepwidth Illustration



Effective Sweepwidth

$$w_e = 2d$$

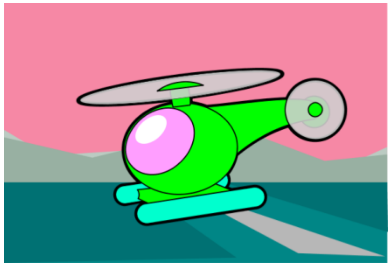
- How do we find *d* ?

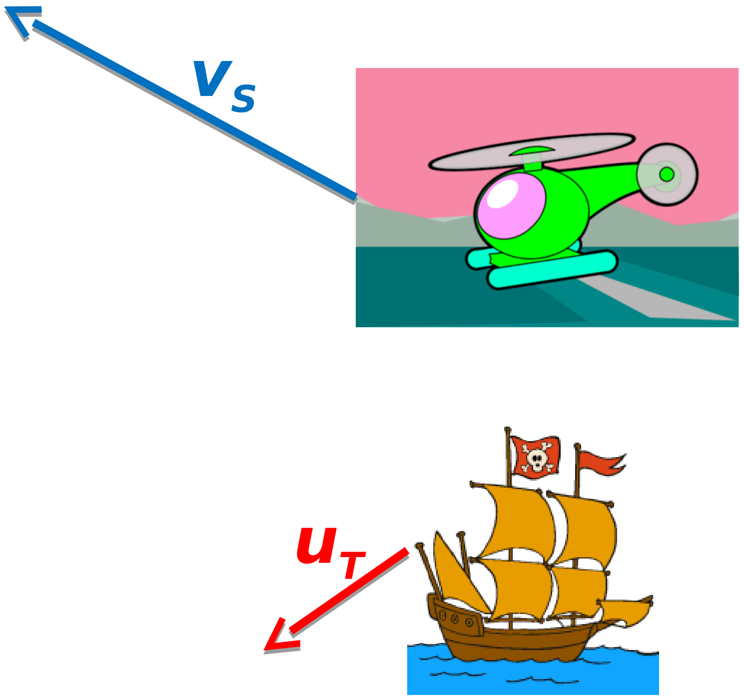
- If you are the target, what course are you taking to evade detection?
  - Assume both searcher and target know values of u, v, R_s, R_t

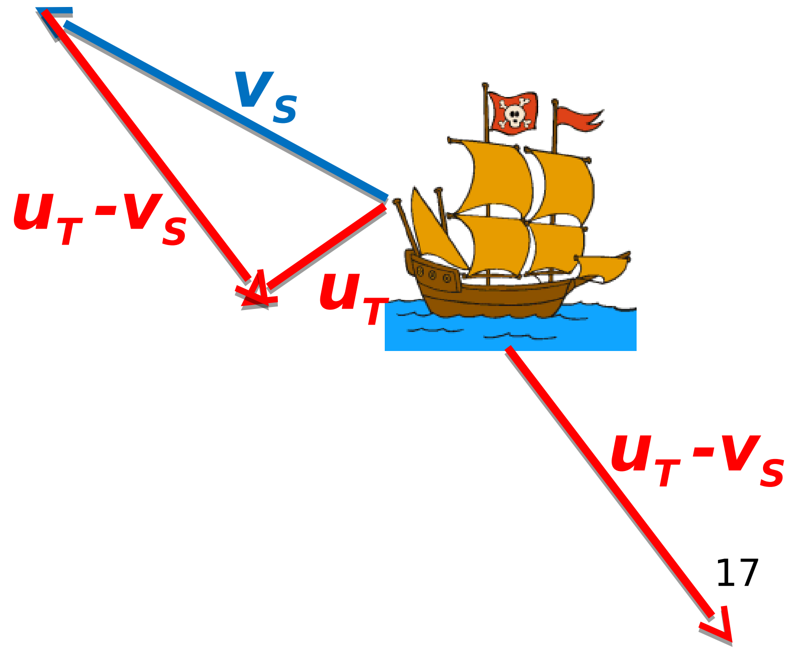
- Target heads in direction to maximize distance at CPA

Motion Relative to Searcher

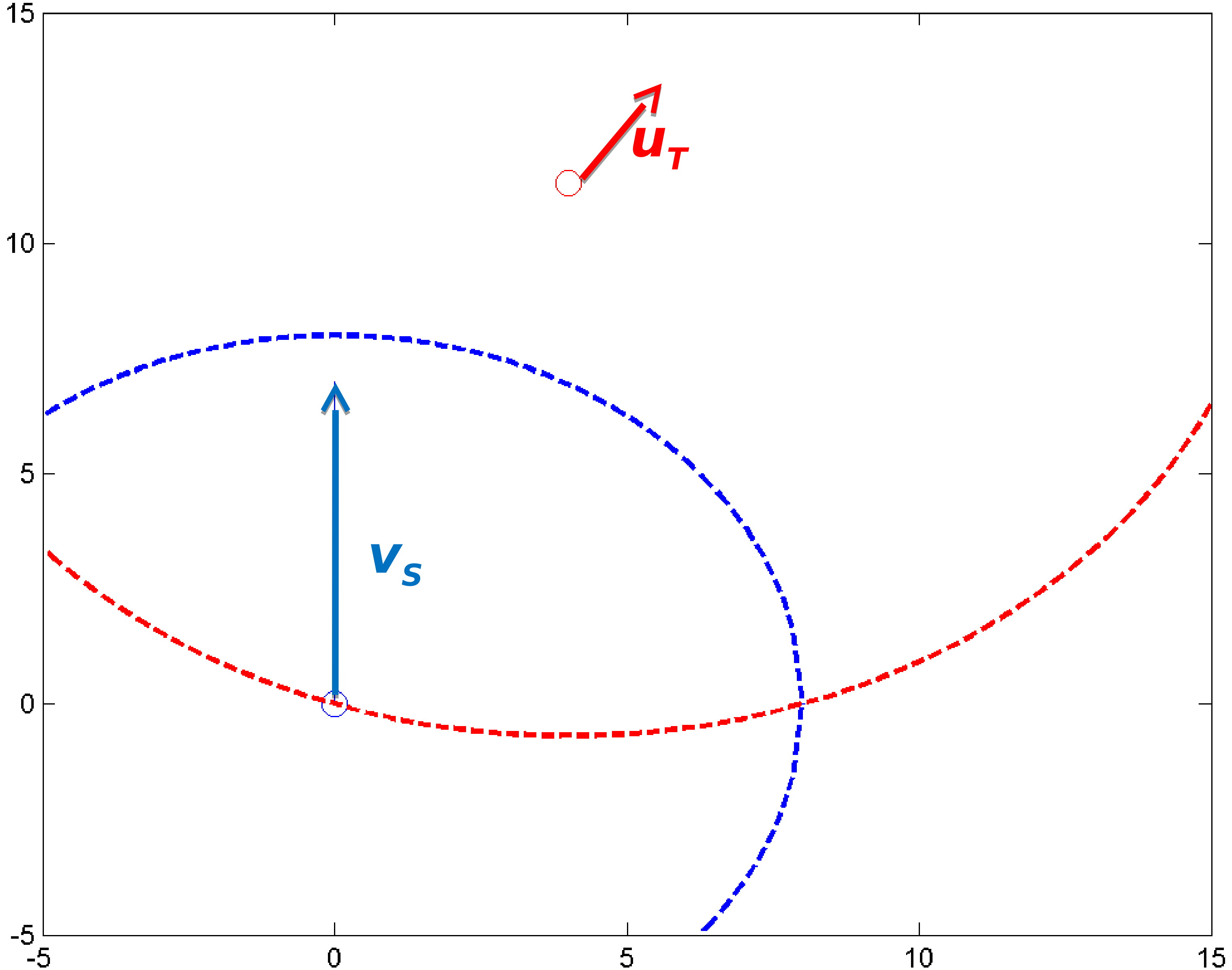
- How Target moves from point of view of Searcher
- View blimp hovering over Searcher. How does Target appear to be moving from point of view of someone on blimp
- Target relative velocity: $\boldsymbol{u_T} - \boldsymbol{v_S}$







Target Motion Relative to Searcher



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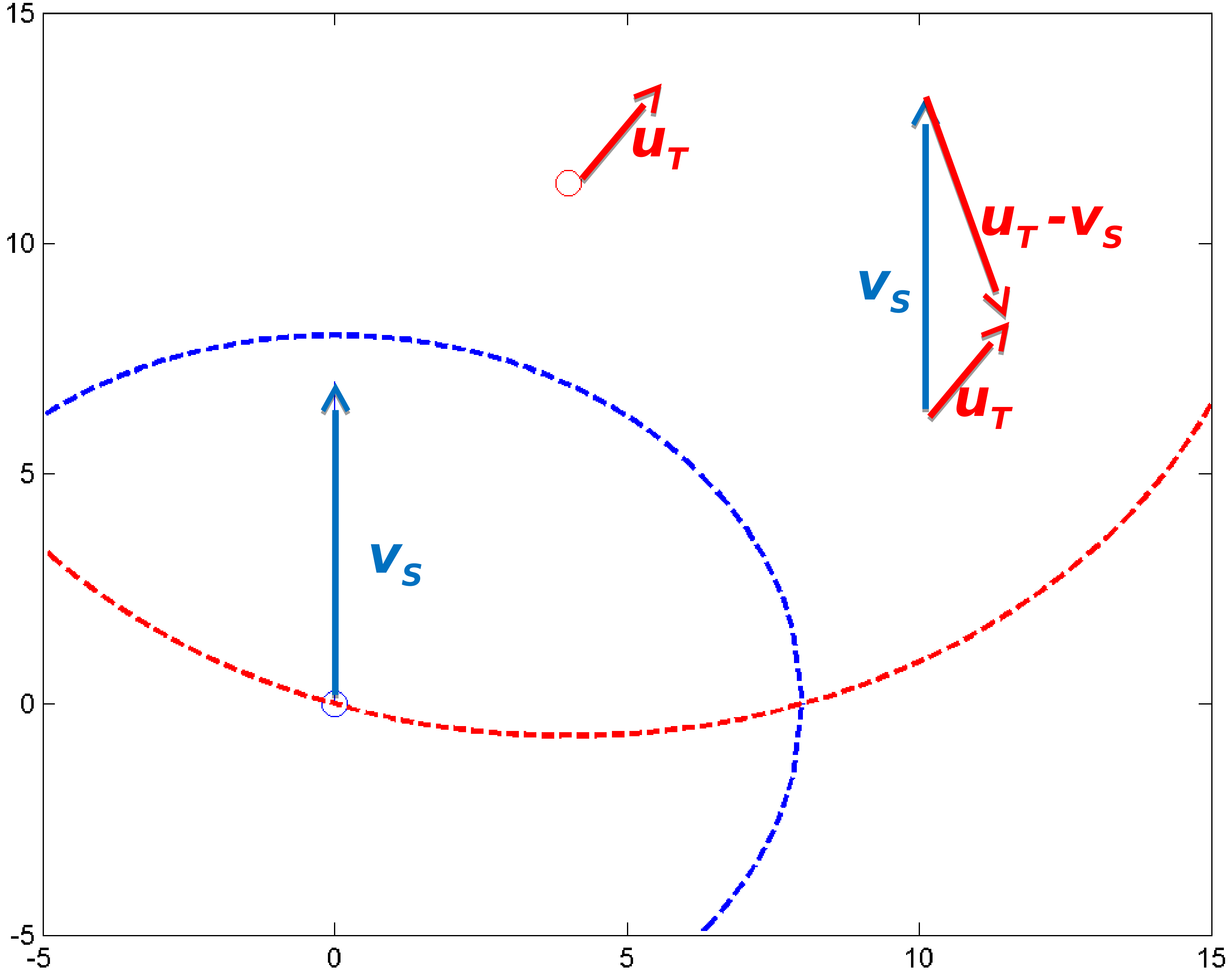

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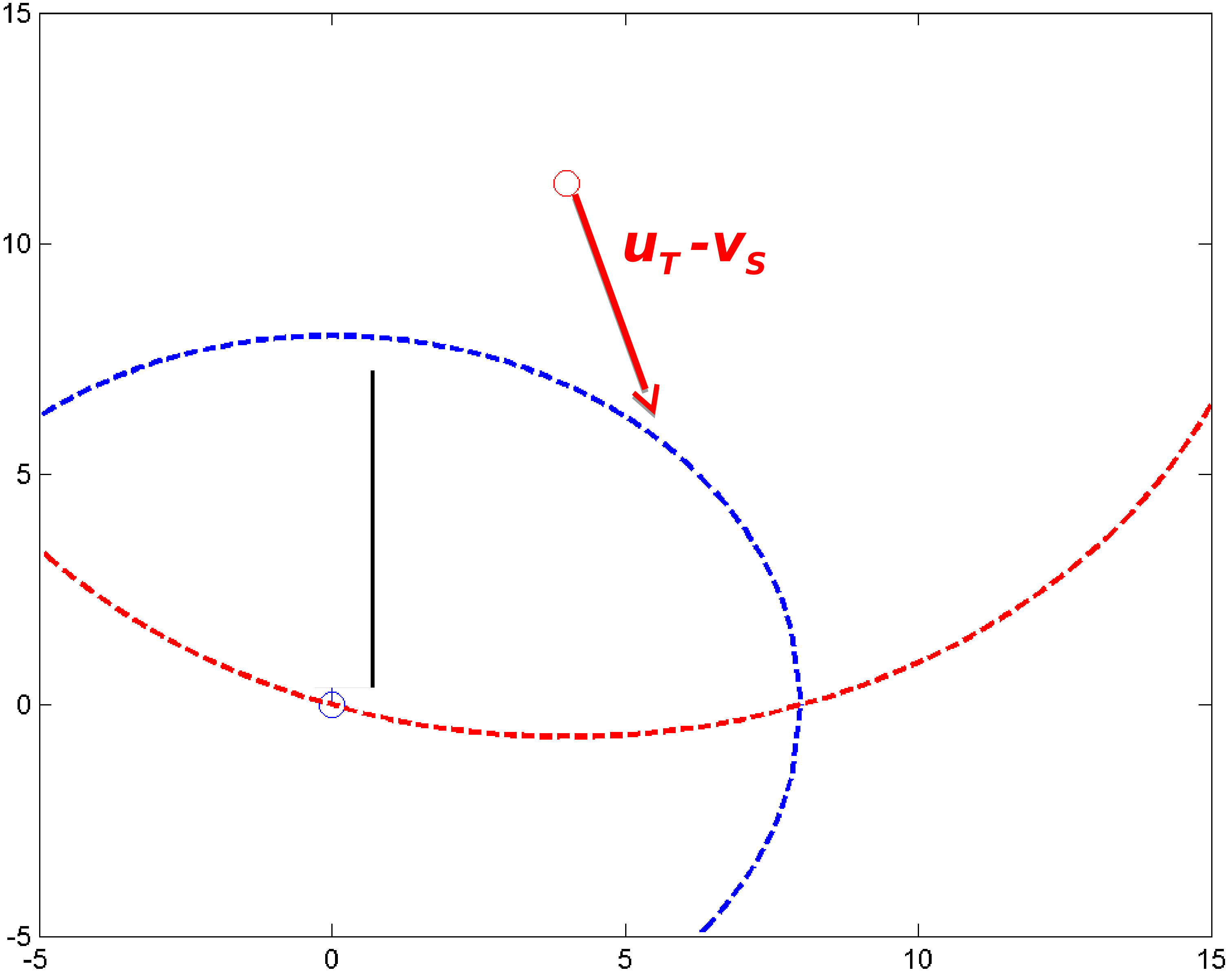

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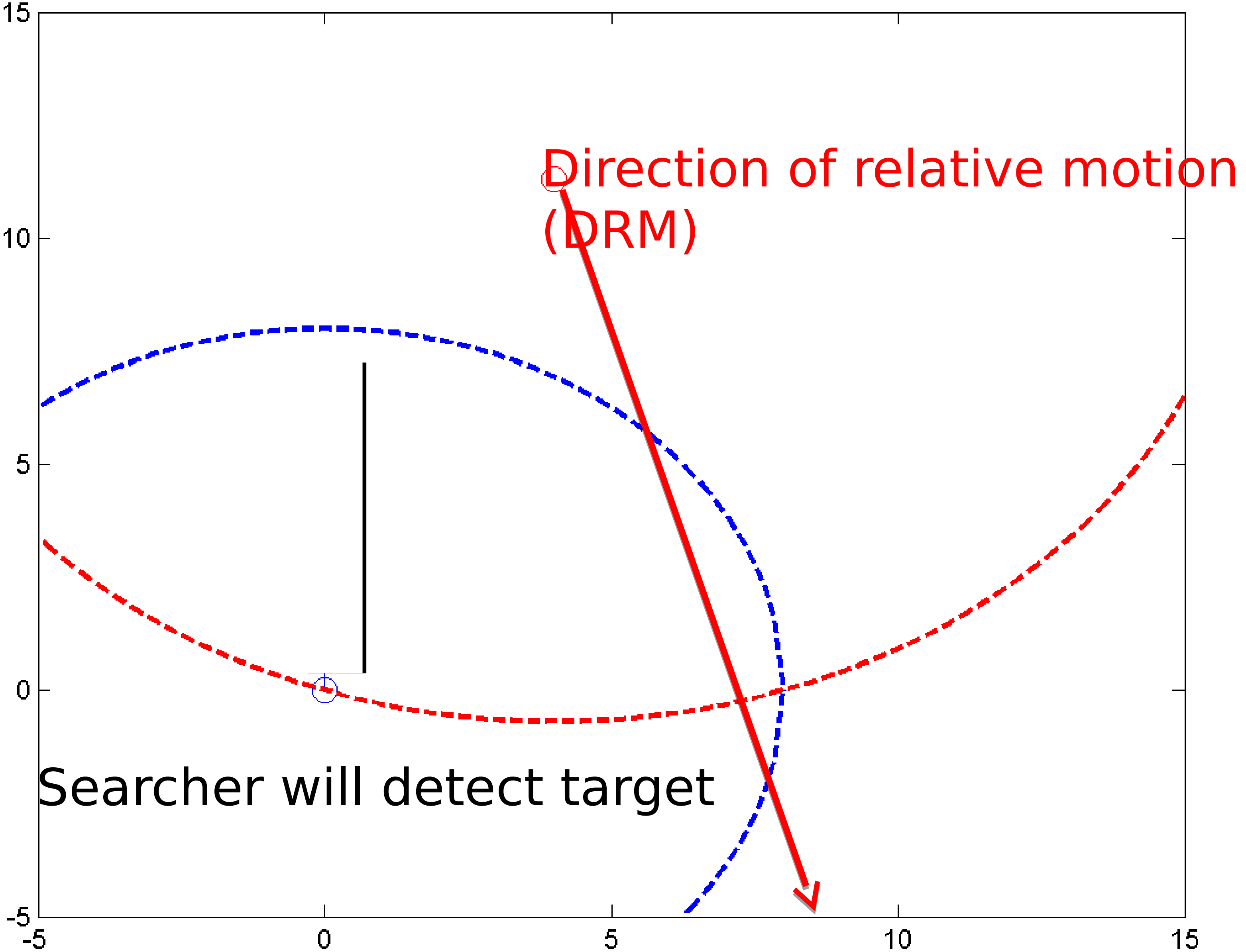

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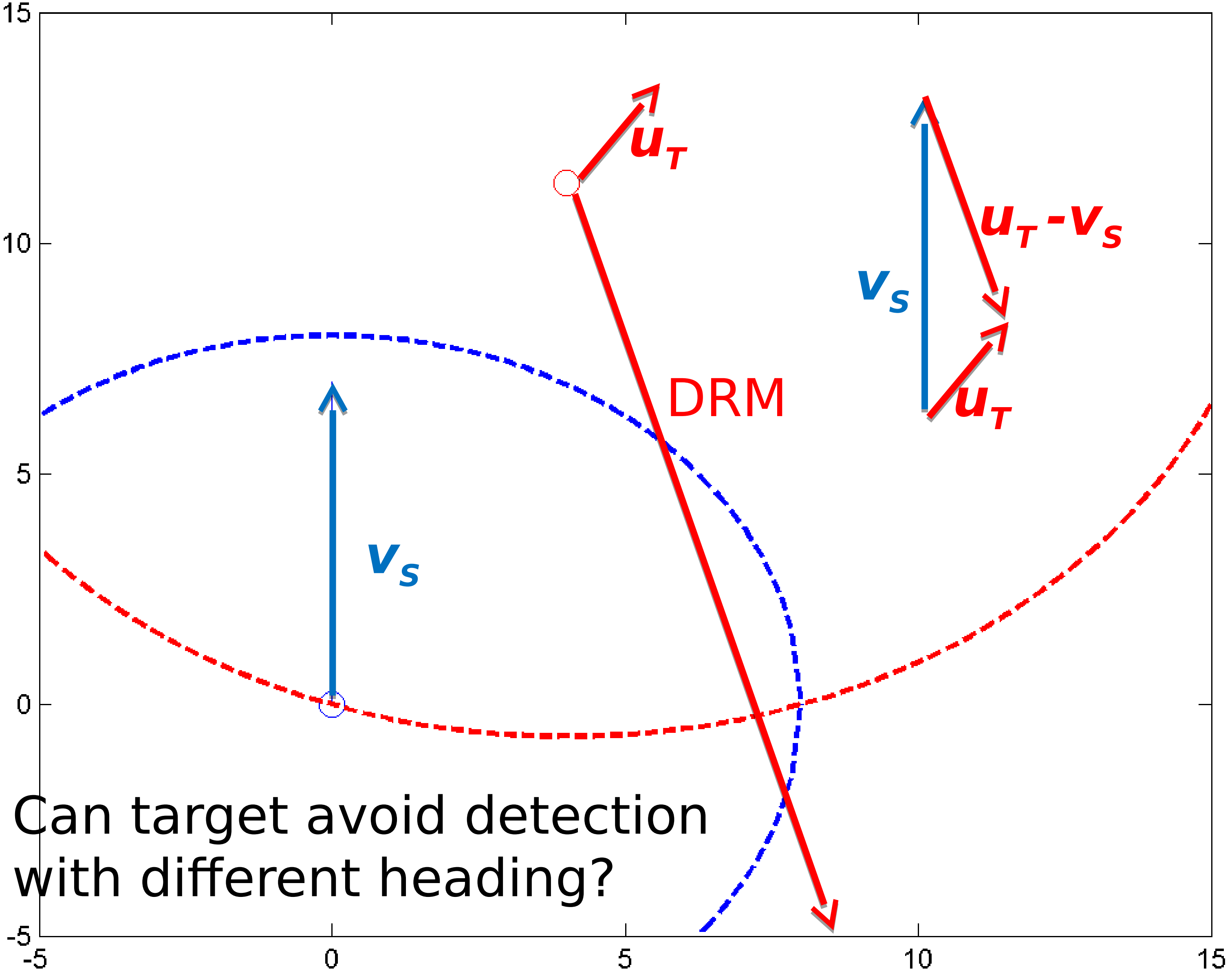

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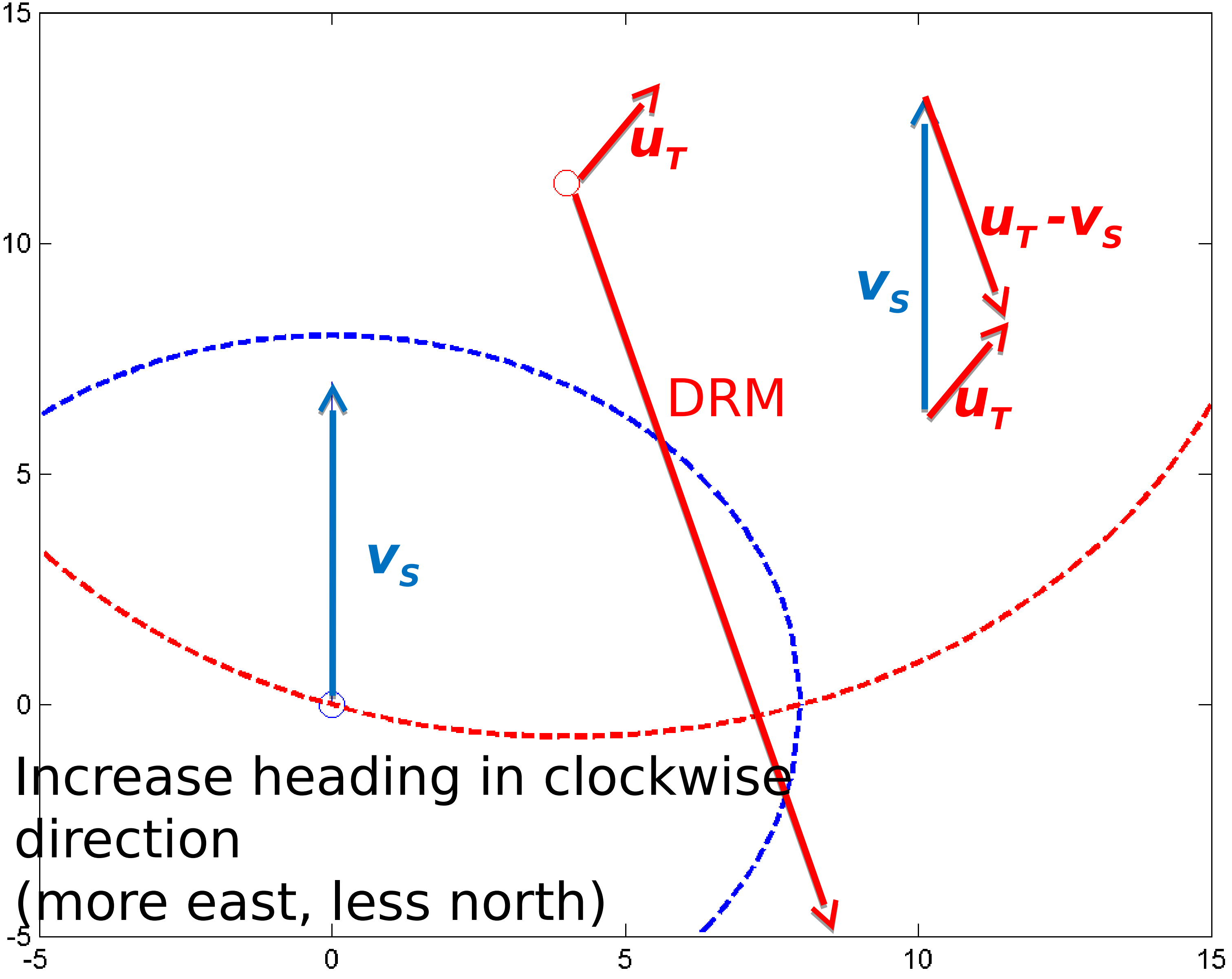

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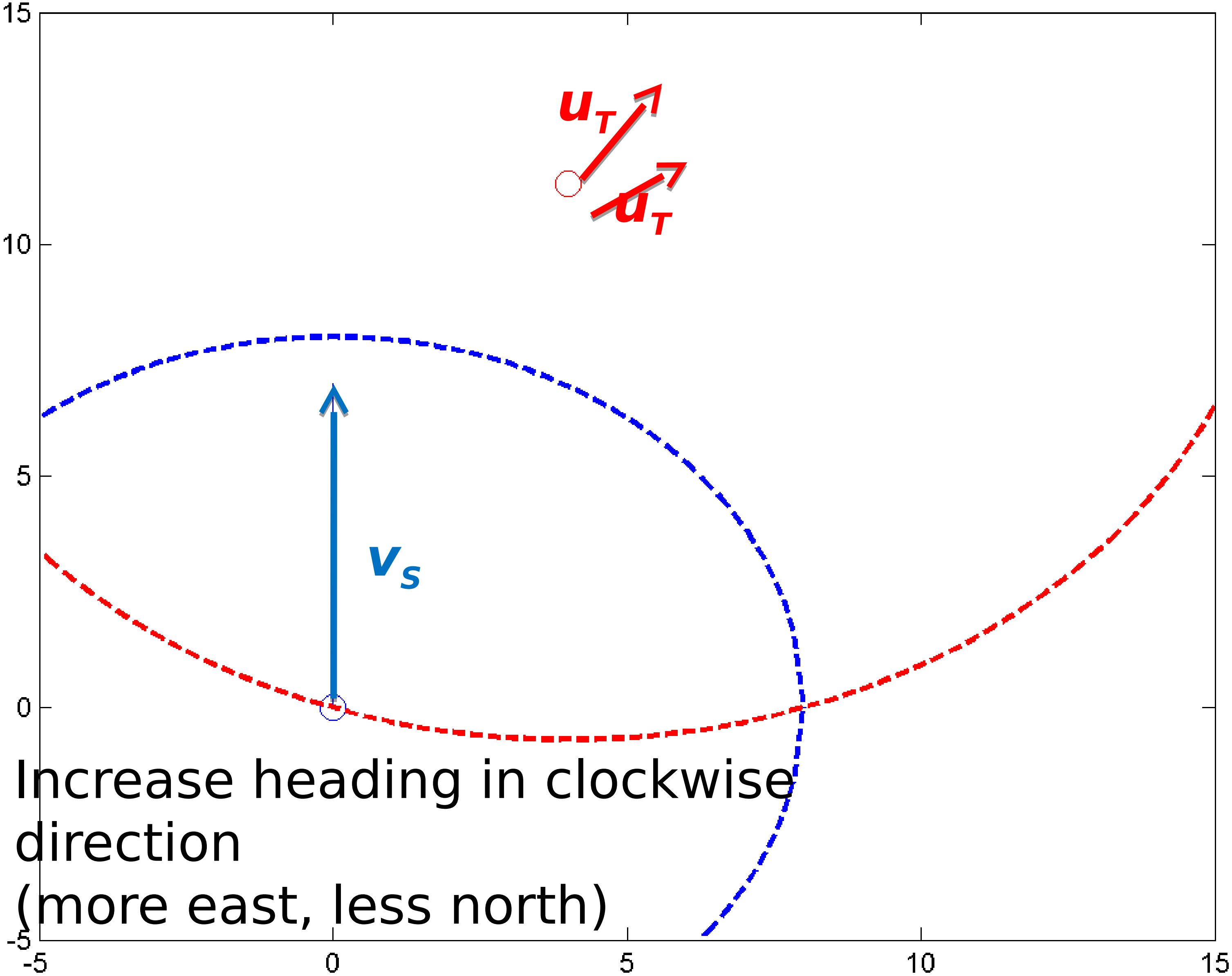
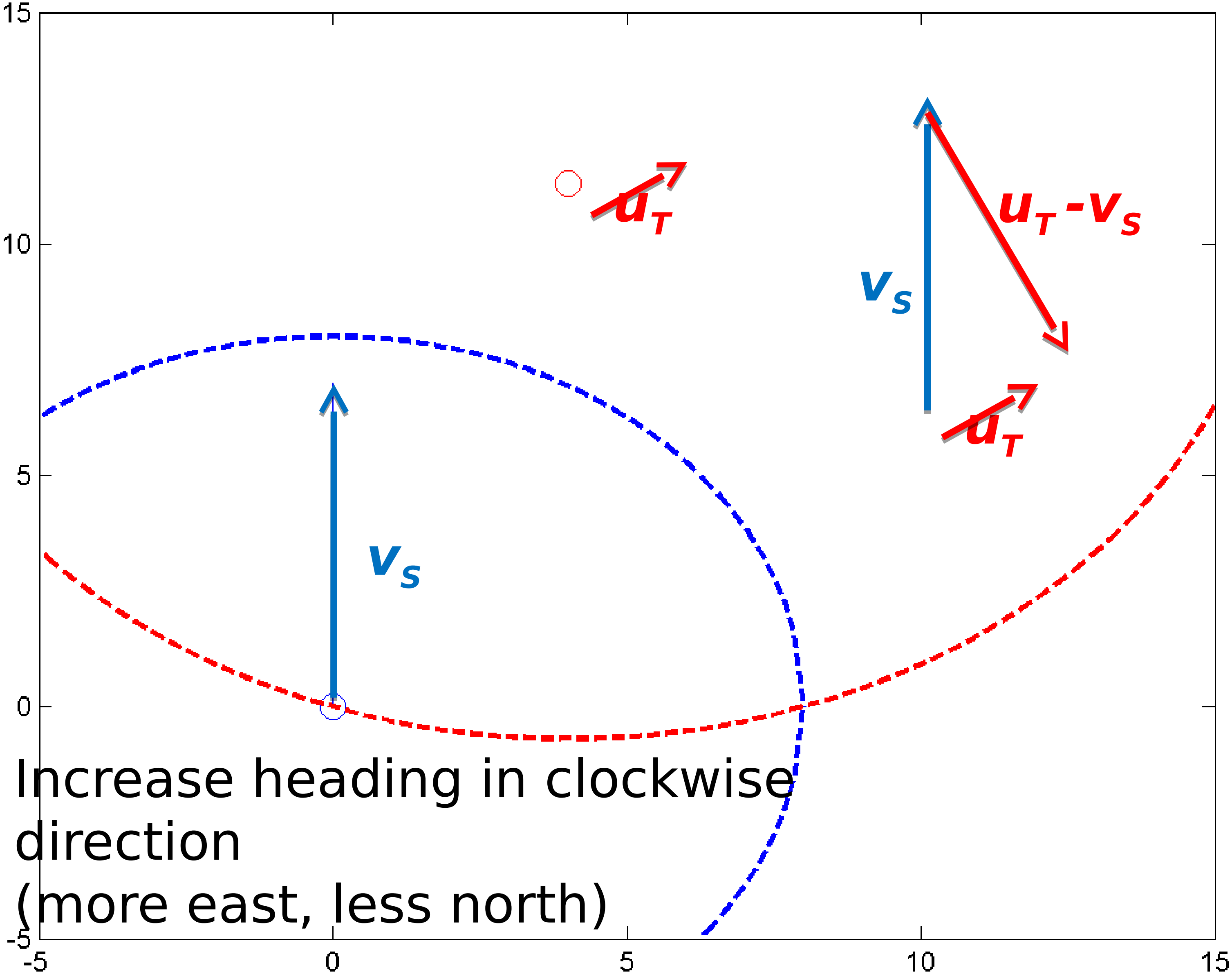


Increase heading in clockwise direction
(more east, less north)

Target Motion Relative to Searcher



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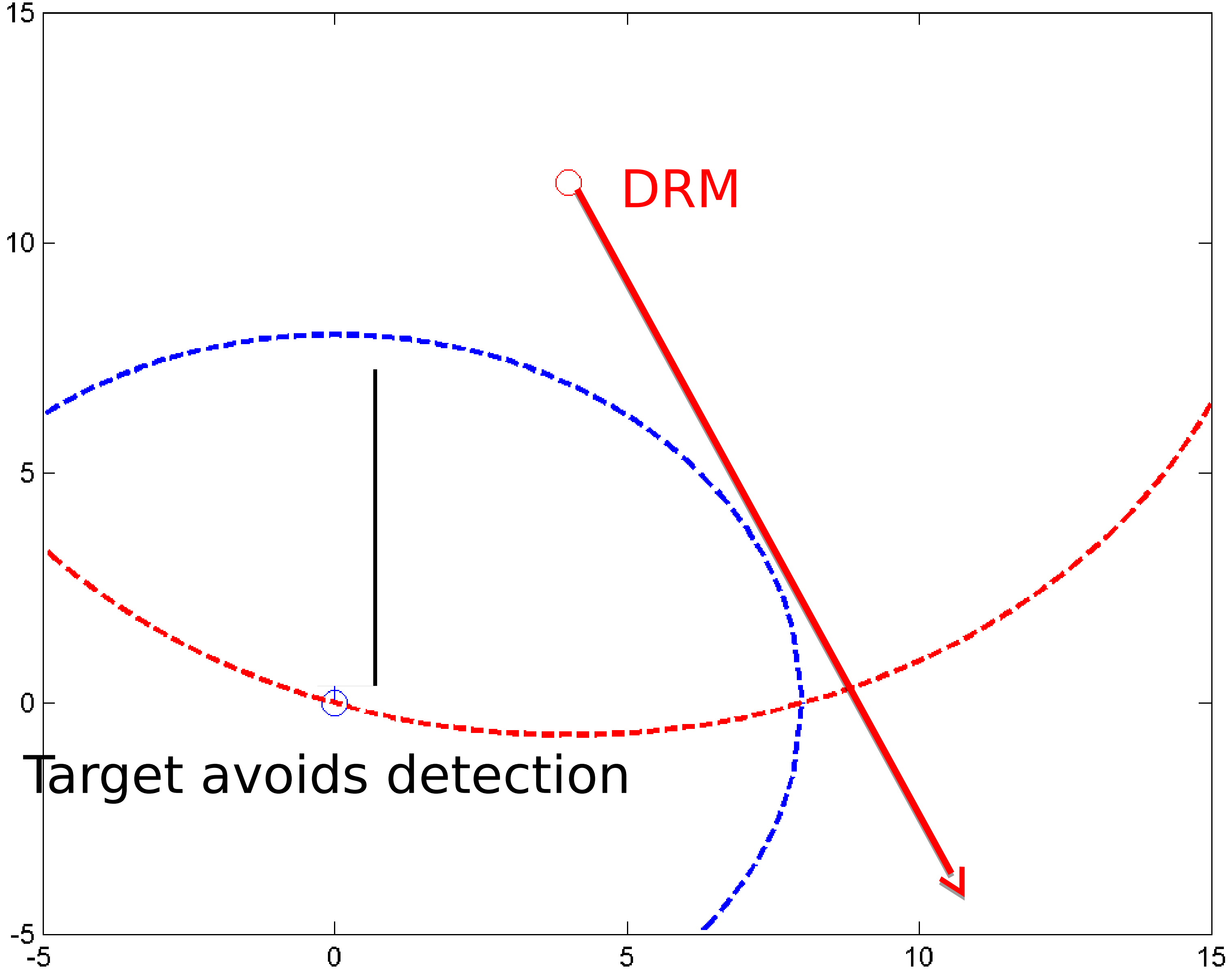


Target's Evasion Heading

$$w_e = 2d$$

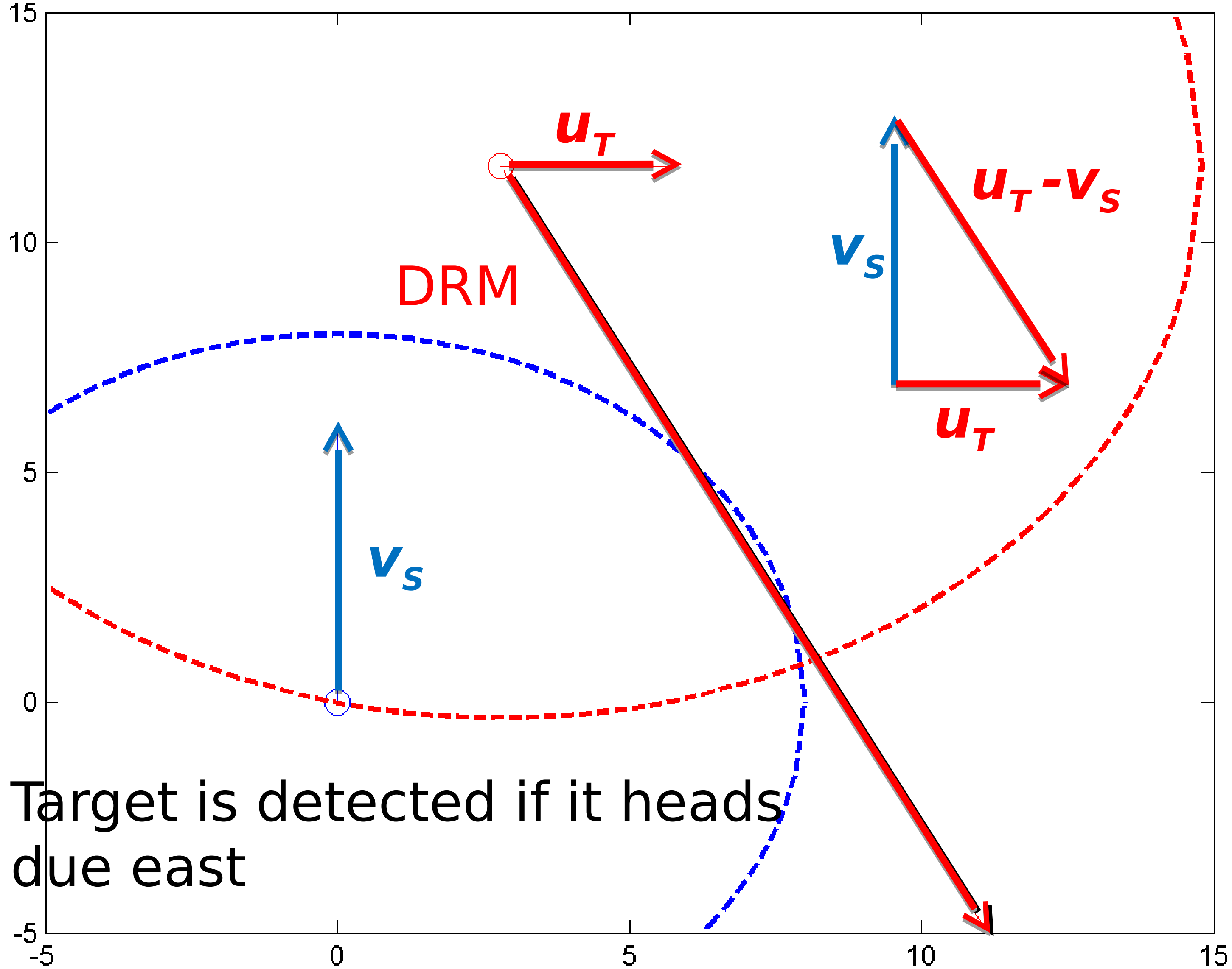
- How do we find *d* ?

- If you are the target, what course are you taking to evade detection?

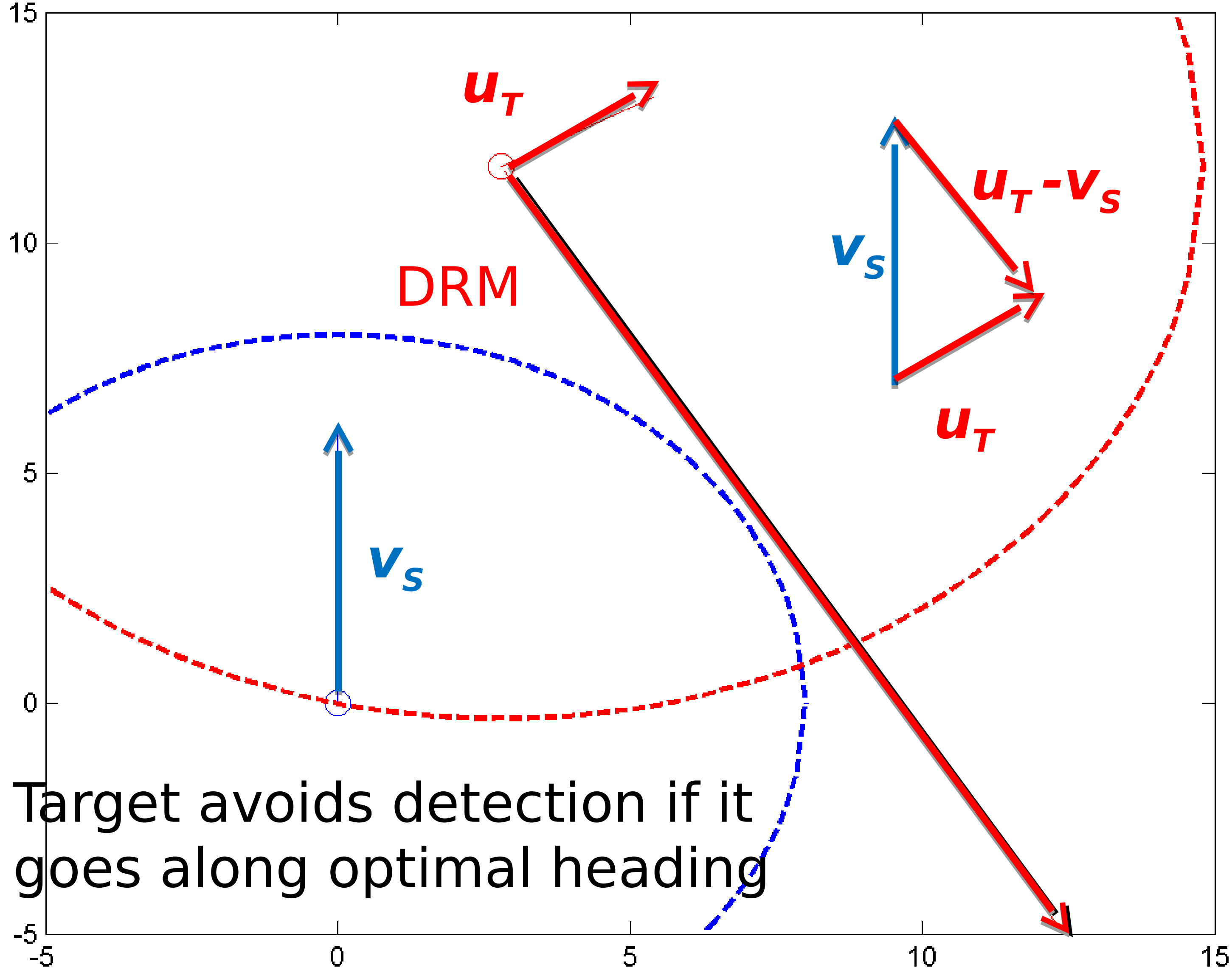
- Target heads in direction to maximize distance at CPA

- Target chooses velocity vector $\boldsymbol{u_T}$ such that $\boldsymbol{u_T}$ is heading away from the searcher and the vectors $\boldsymbol{u_T}$ and $\boldsymbol{u_T} - \boldsymbol{v_S}$ are perpendicular

Target's Evasion Heading

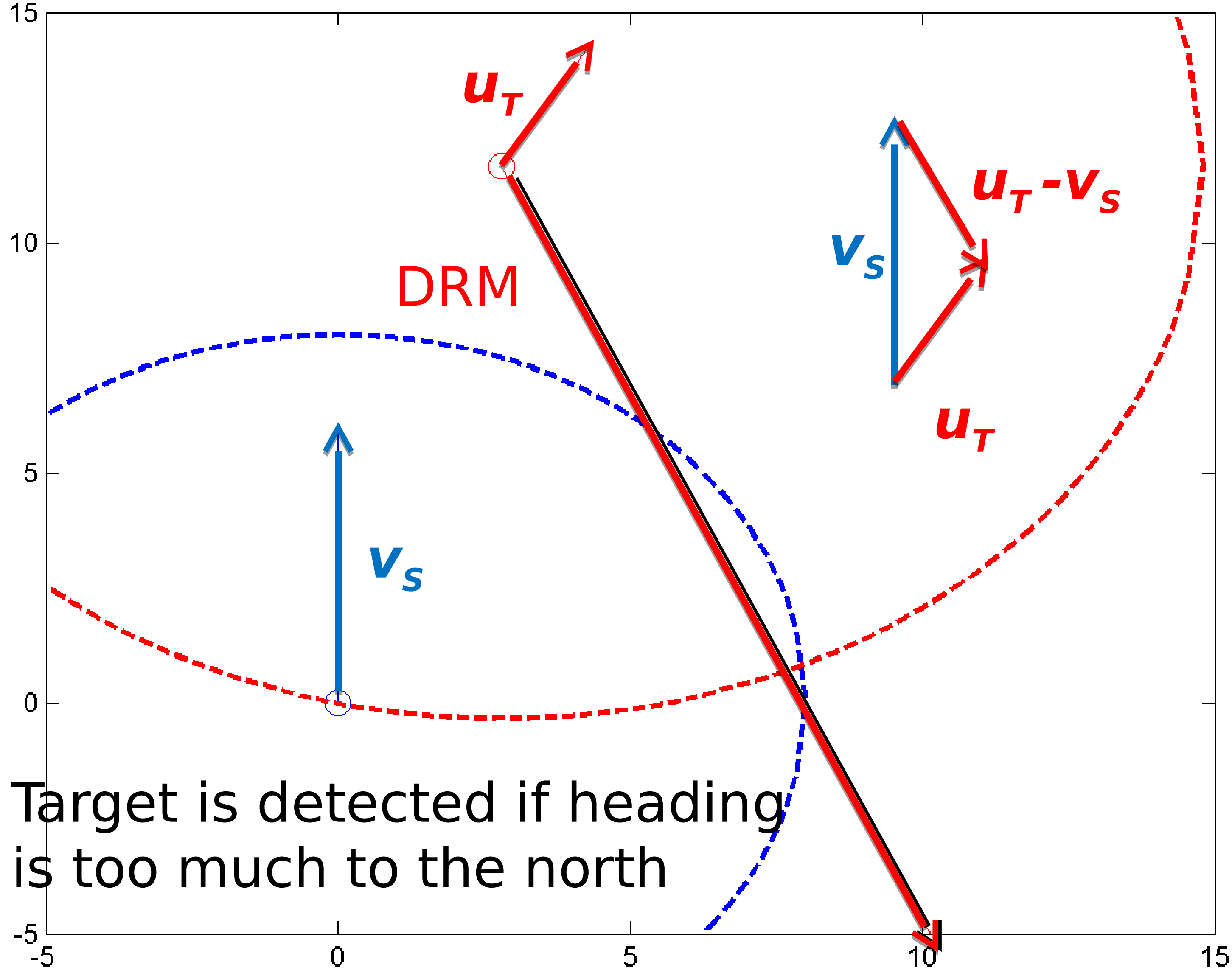


Target's Evasion Heading



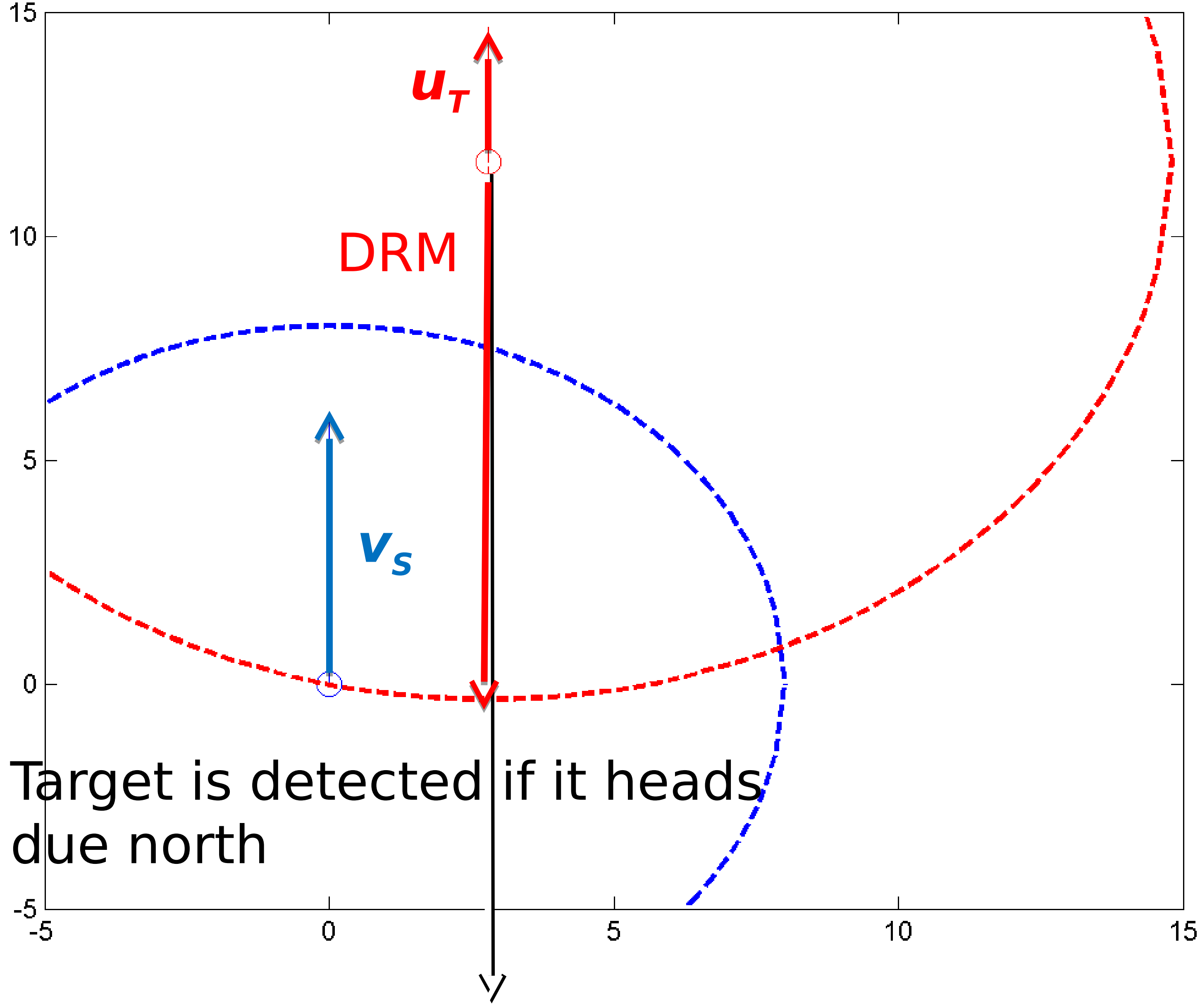
Target avoids detection if it goes along optimal heading

Target’s Evasion Heading



Target is detected if heading
is too much to the north

Target's Evasion Heading



Target is detected if it heads due north

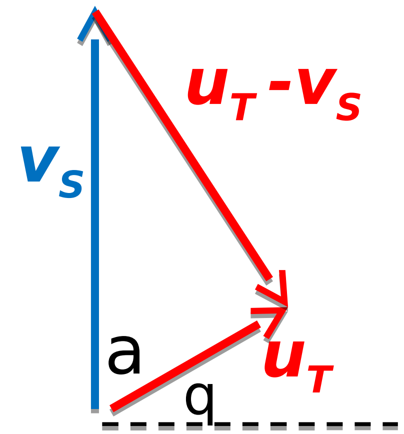
Effective Sweepwidth

$$w_e = 2d$$

- How do we find *d* ?

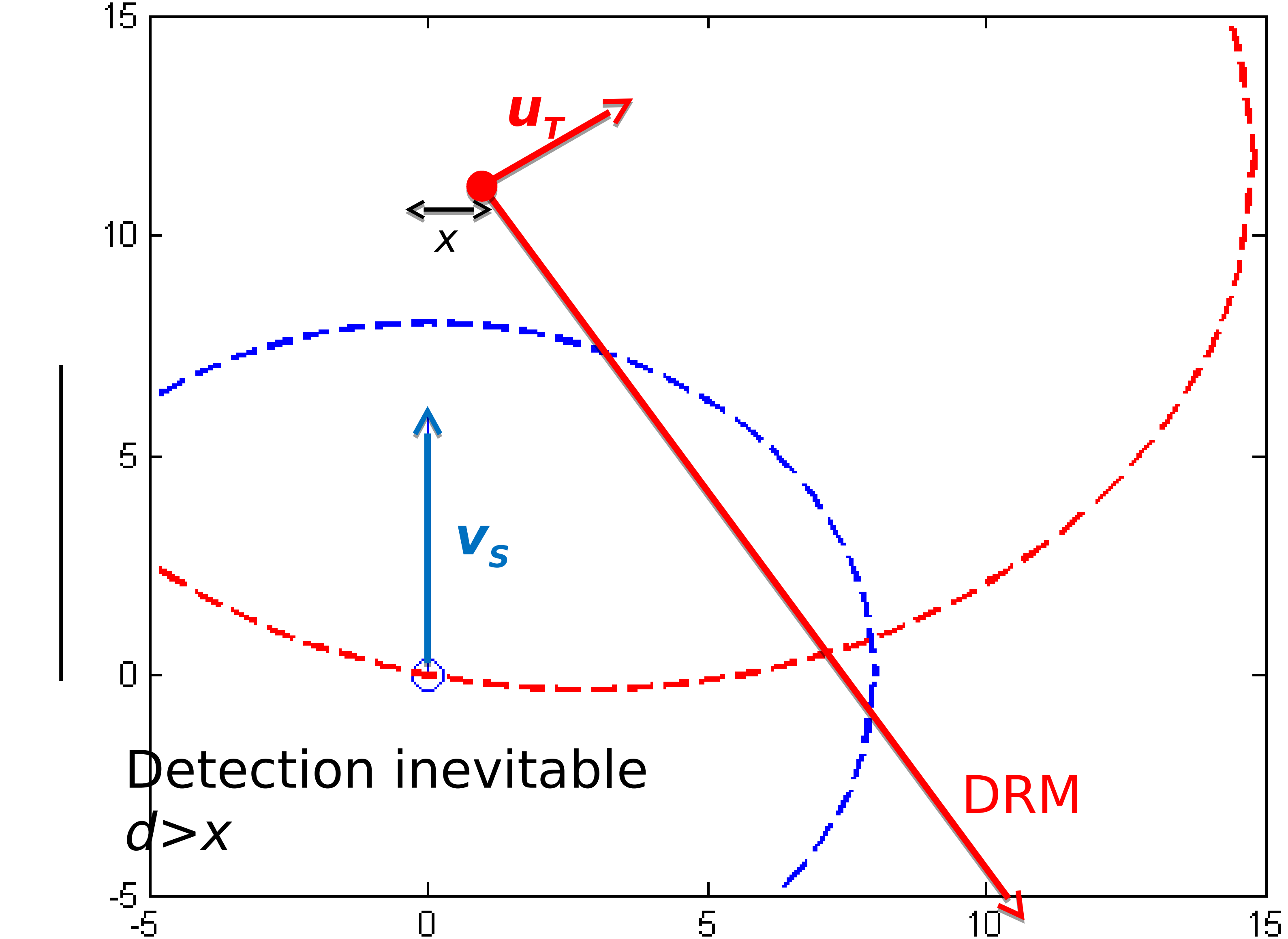
- Target chooses velocity vector $\boldsymbol{u_T}$ such that $\boldsymbol{u_T}$ is heading away from the searcher and the vectors $\boldsymbol{u_T}$ and $\boldsymbol{u_T} - \boldsymbol{v_S}$ are perpendicular

$$\alpha = \cos^{-1}\left(\frac{u}{v}\right) \qquad \theta = \pi/2 - \alpha$$



- Given optimal heading, can the target avoid detection?

Determination of *d*



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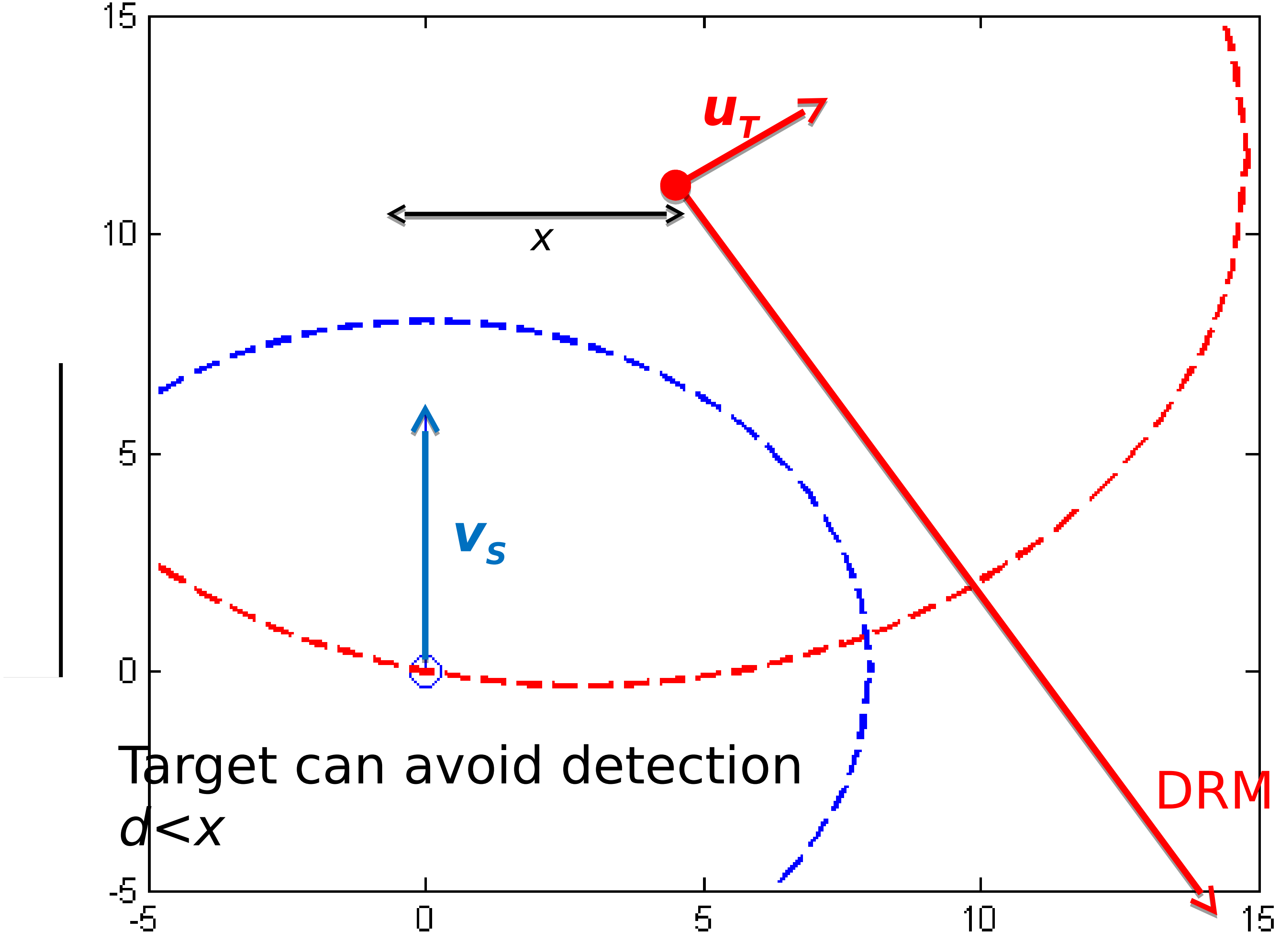
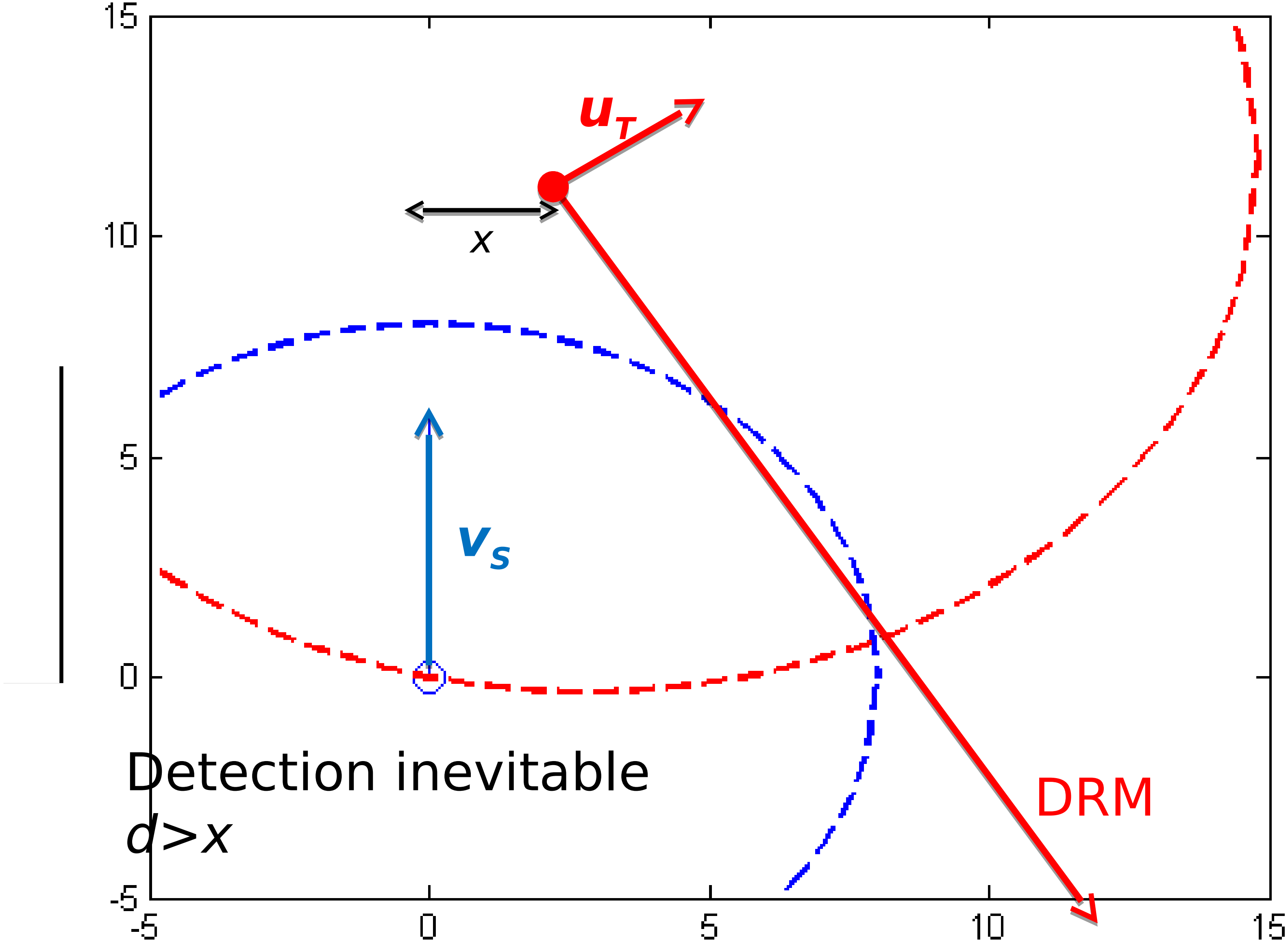


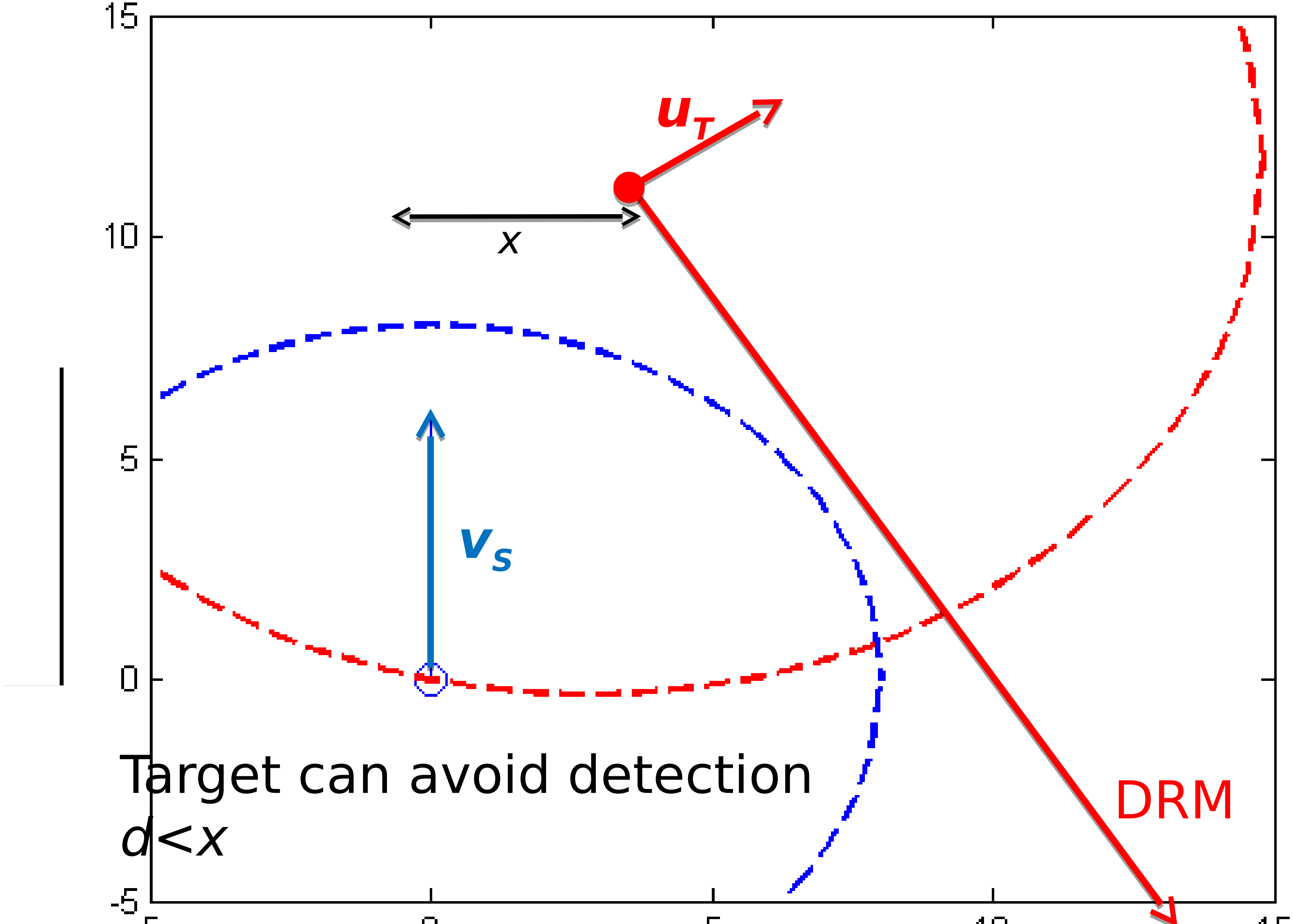
Target can avoid detection

$d<x$

Determination of *d*



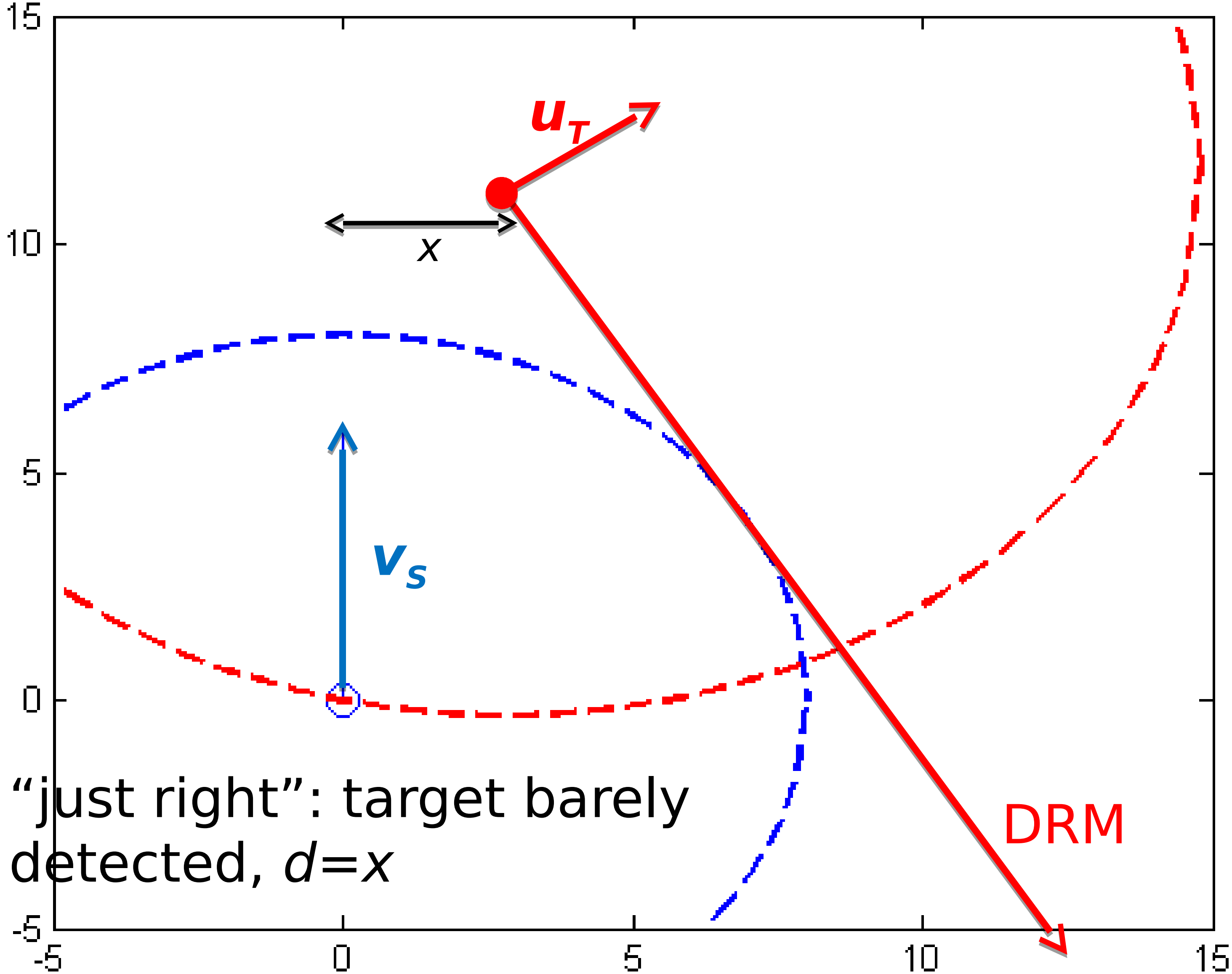
Determination of *d*



Target can avoid detection

$d<x$

Determination of *d*



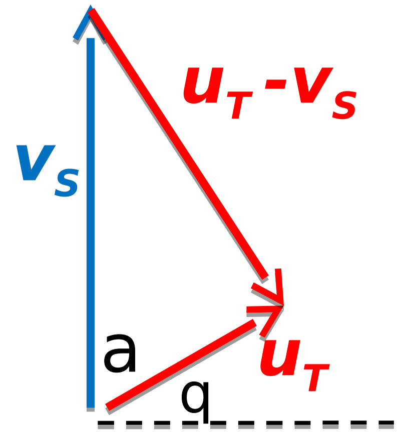
“just right”: target barely detected, d=x

Effective Sweepwidth

$$w_e = 2d$$

- We know the optimal heading of target

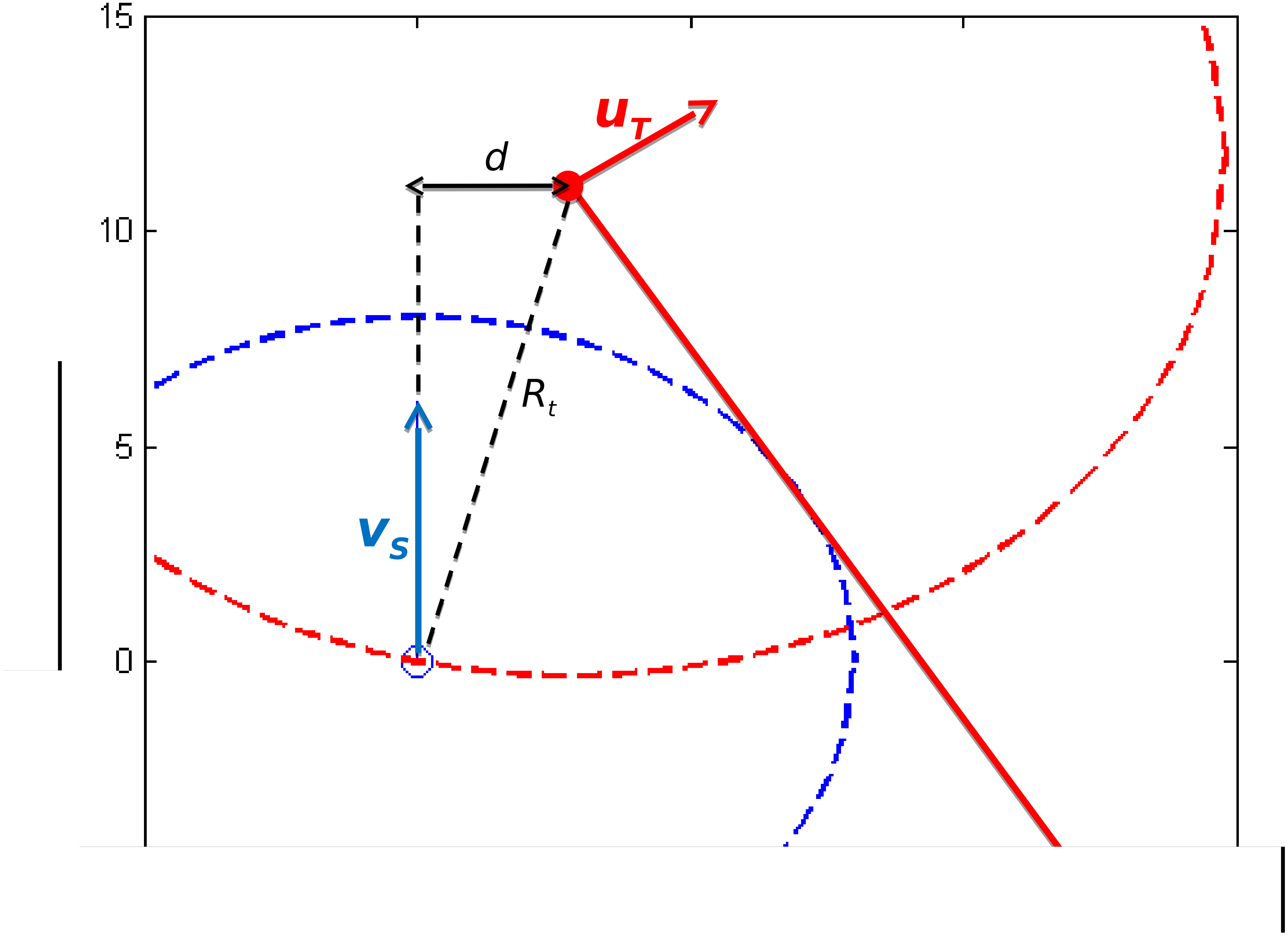
$$\alpha = \cos^{-1}\left(\frac{u}{v}\right) \qquad \theta = \pi/2 - \alpha$$



- On the previous slides we illustrated pictorially what conditions *d* must satisfy?
  - DRM tangent to searcher sensor circle

- Computing *d* explicitly requires going through the geometry/trigonometry

Determination of *d*



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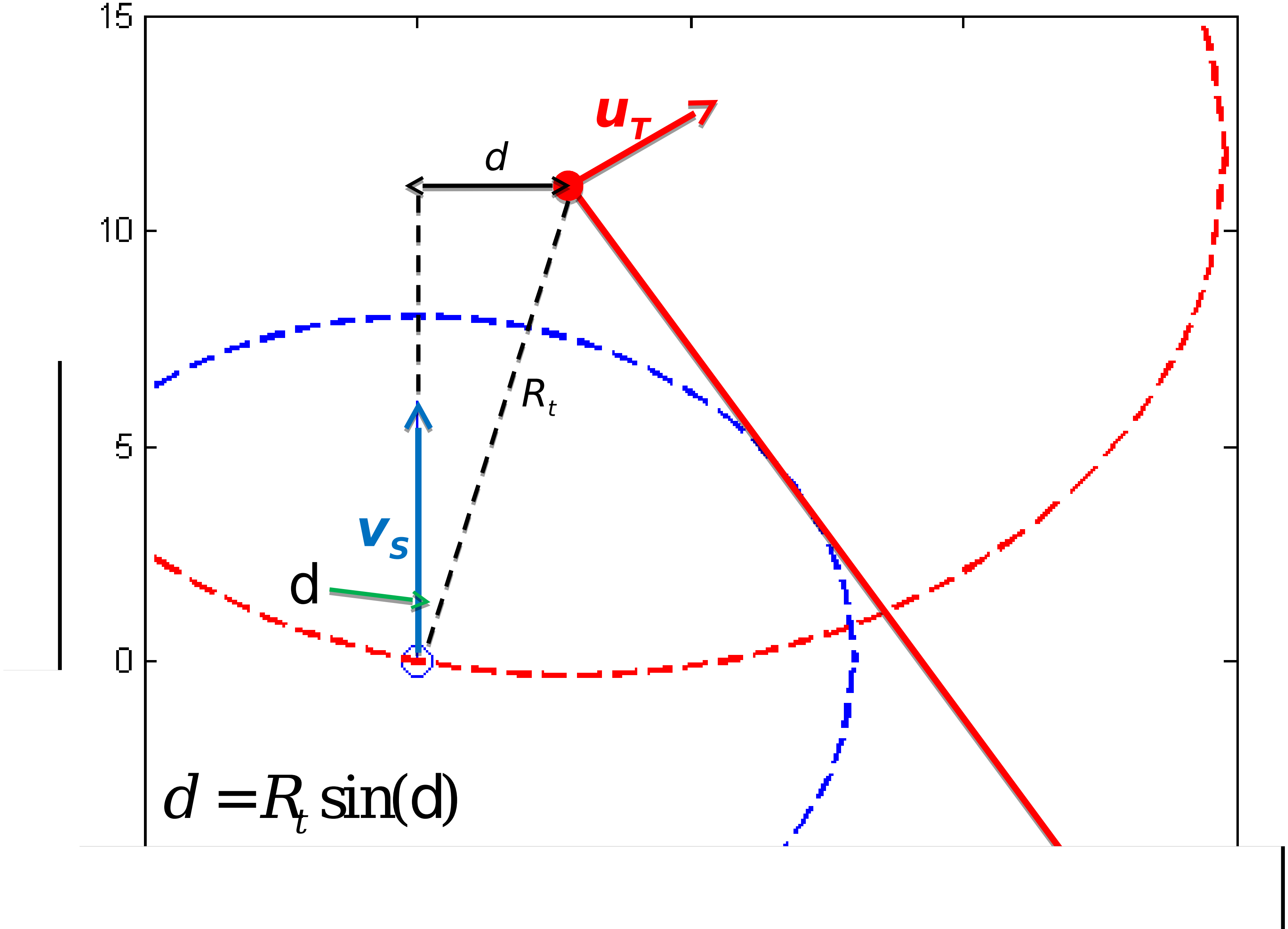
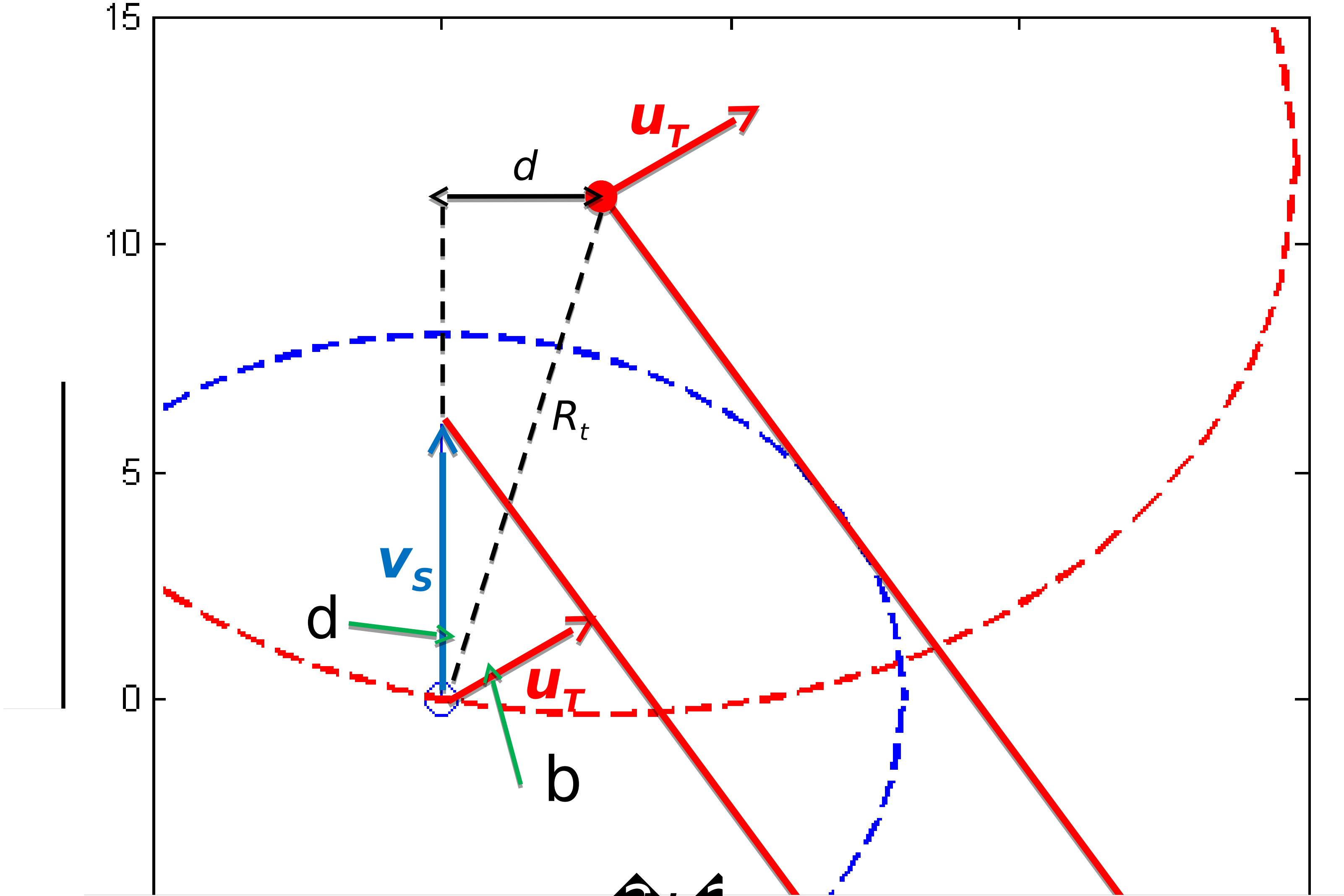
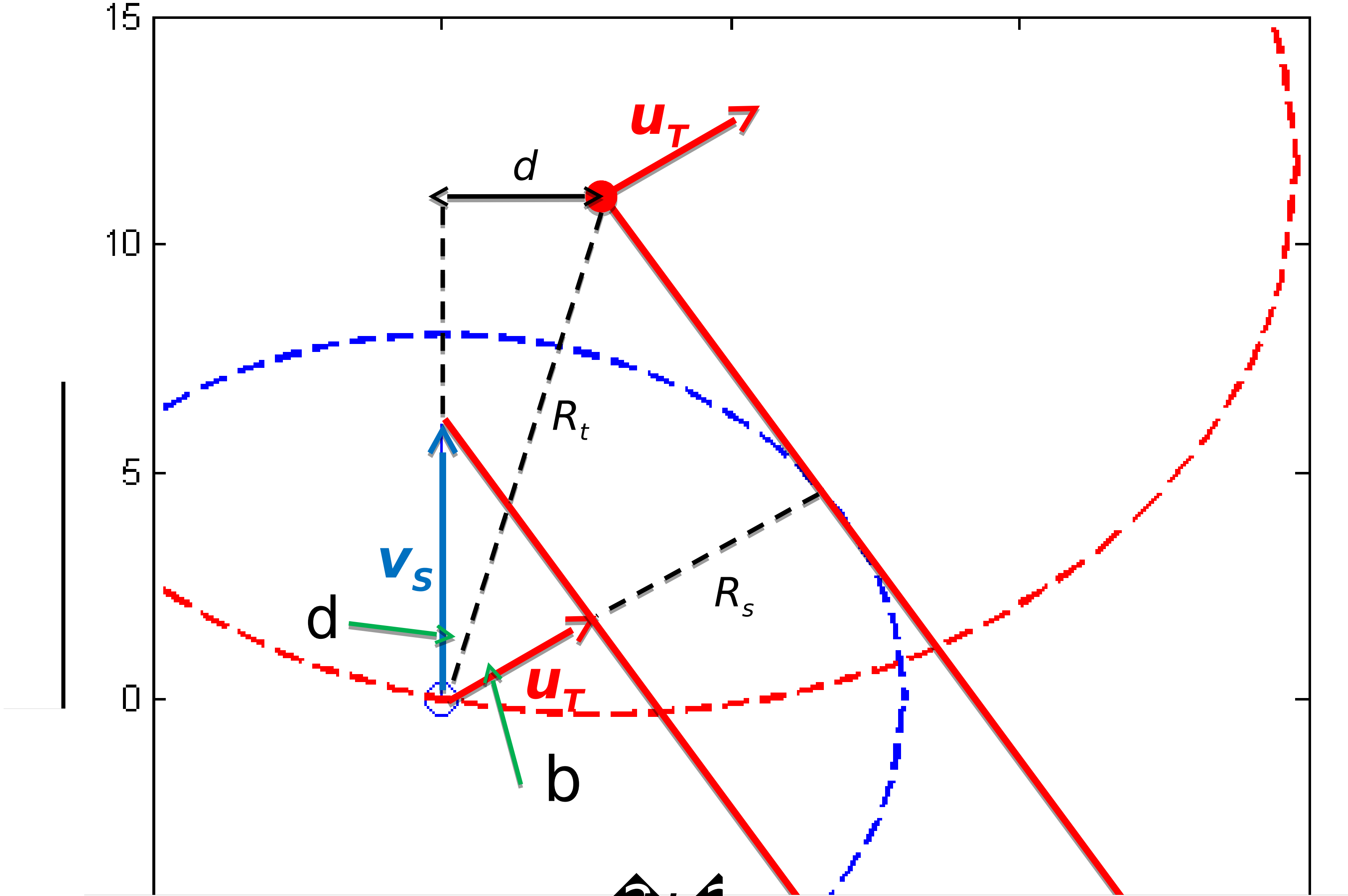


Determination of *d*



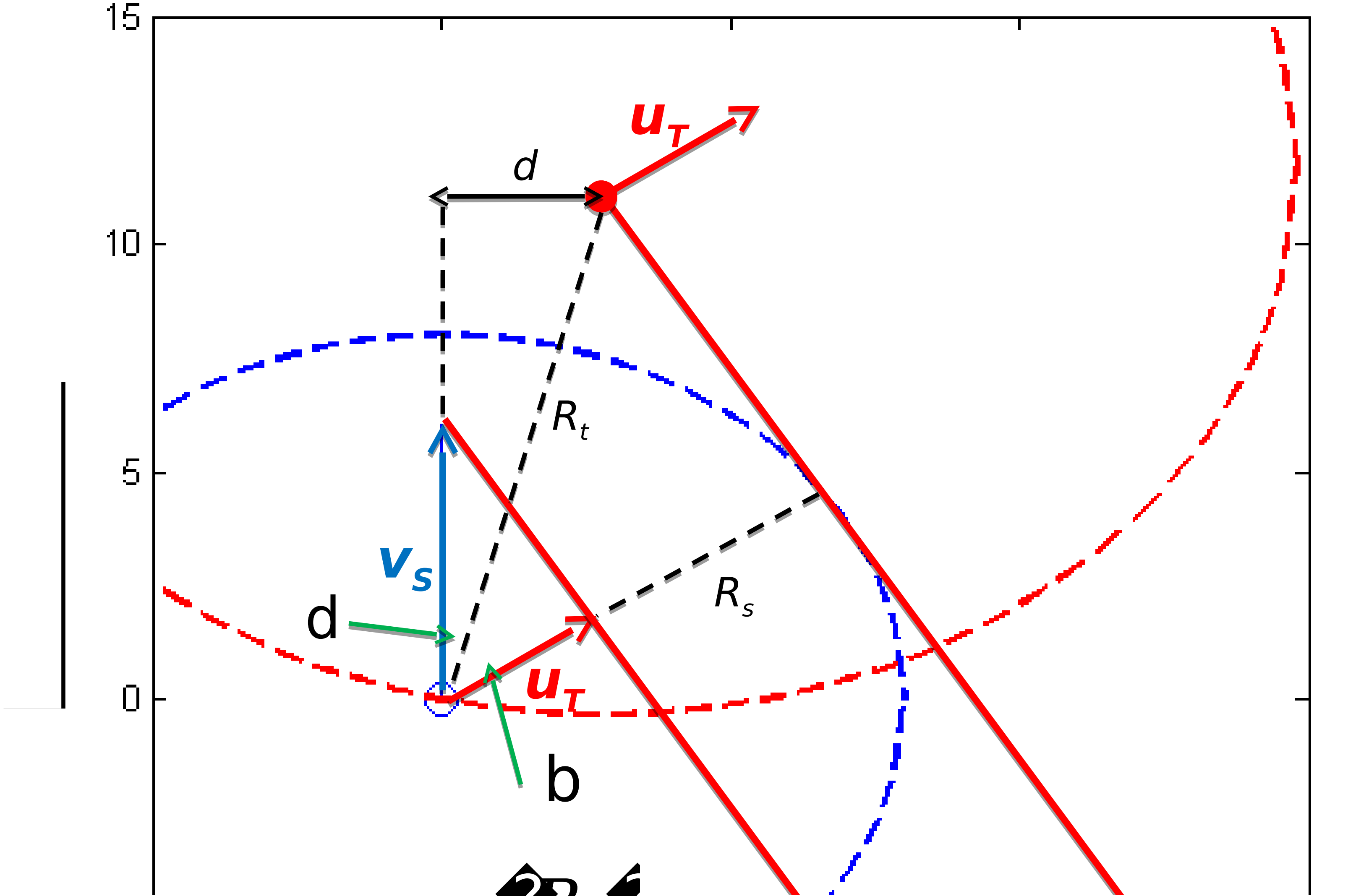
$$d+b=a=\cos^{-1}\left(\frac{u}{v}\right)$$

Determination of *d*



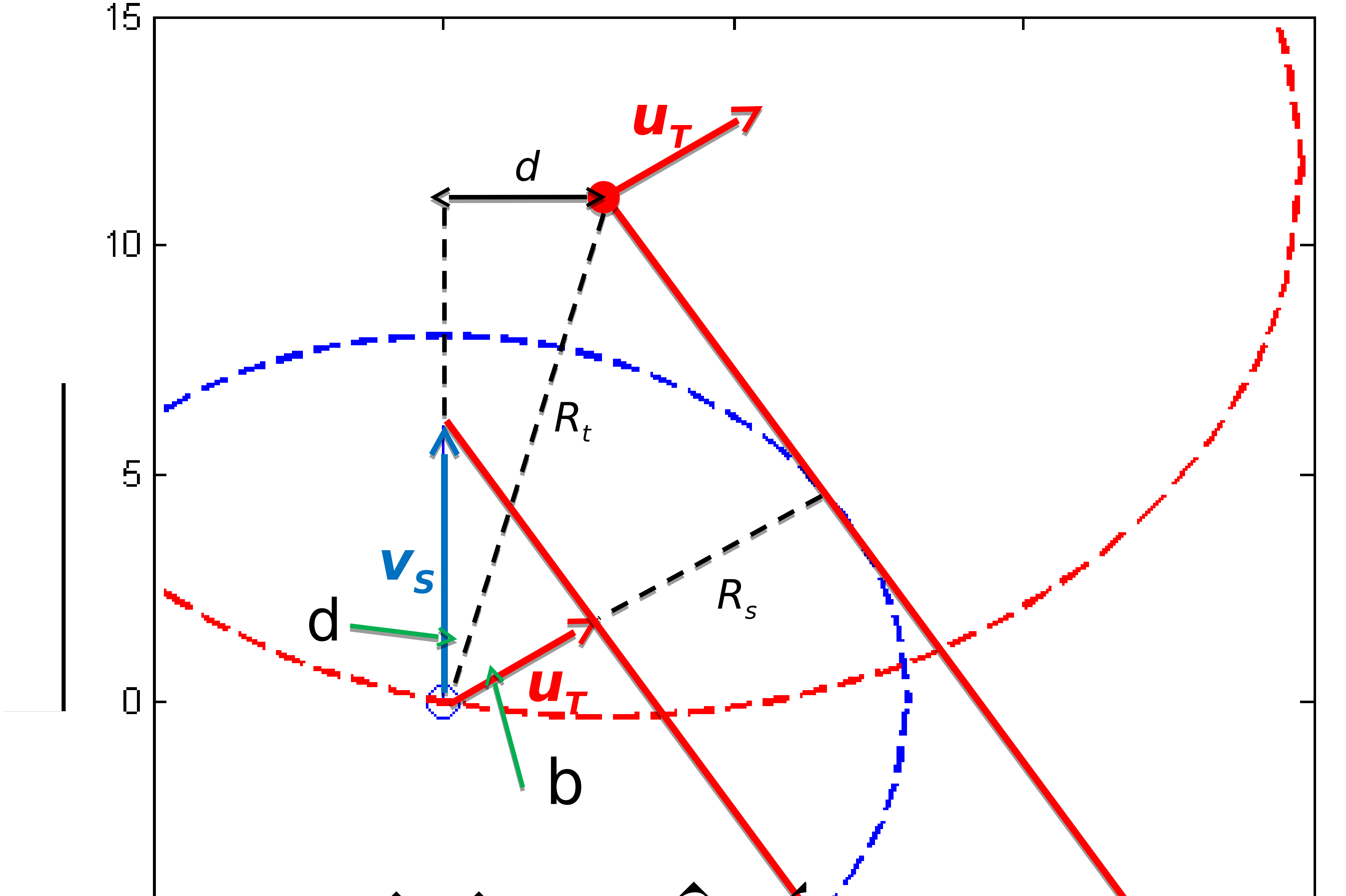
$$d + b = a = \cos^{-1}\left(\frac{u}{v}\right)$$

Determination of *d*



$$b = \cos^{-1}\left(\frac{R_s}{R_t}\right)$$

Determination of *d*



$$d = a - b = \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)$$

Summary

$w_e = 2d \qquad d = R_t \sin(d) \qquad d = a - b = \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)$

$$\boxed{w_e = 2R_t \sin\left(\cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right)}$$

- Only valid for
  - v > u : otherwise target always escapes and w_e=0
  - R_t > R_s : otherwise counter-detections worthless and w_e=2R_s
- Also only valid for $a > b \Leftrightarrow \frac{R_s}{R_t} > \frac{u}{v}$
- If $\frac{R_s}{R_t} < \frac{u}{v}$ target has sufficient speed to escape and w_e=0

Numerical Example

- $R_s = 8$
- $R_t = 12$
- $v = 9$
- u = 3

- No countermeasures: sweepwidth w= $2R_s$ = 16nm

- With counter-detection

$$w_e = 2R_t \sin\left(\cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right) = 9.12\text{nm}$$

- If $\frac{R_s}{R_t} < \frac{u}{v}$ then w_e=0
  - E.g., target increase speed $\frac{R_s}{R_t} < \frac{u}{v} \Rightarrow \frac{vR_s}{R_t} < u \Rightarrow u > 6$

Target Wishes to Be Detected

- In the previous analysis, at counter-detection the target chose a heading to evade the target and (hopefully) escape detection
  - Maximize distance at CPA

- What if target wants to be detected?
  - E.g, rescued

- Target might be shooter closing in for attack
  - R_s represents firing distance for target

Effective Sweepwidth

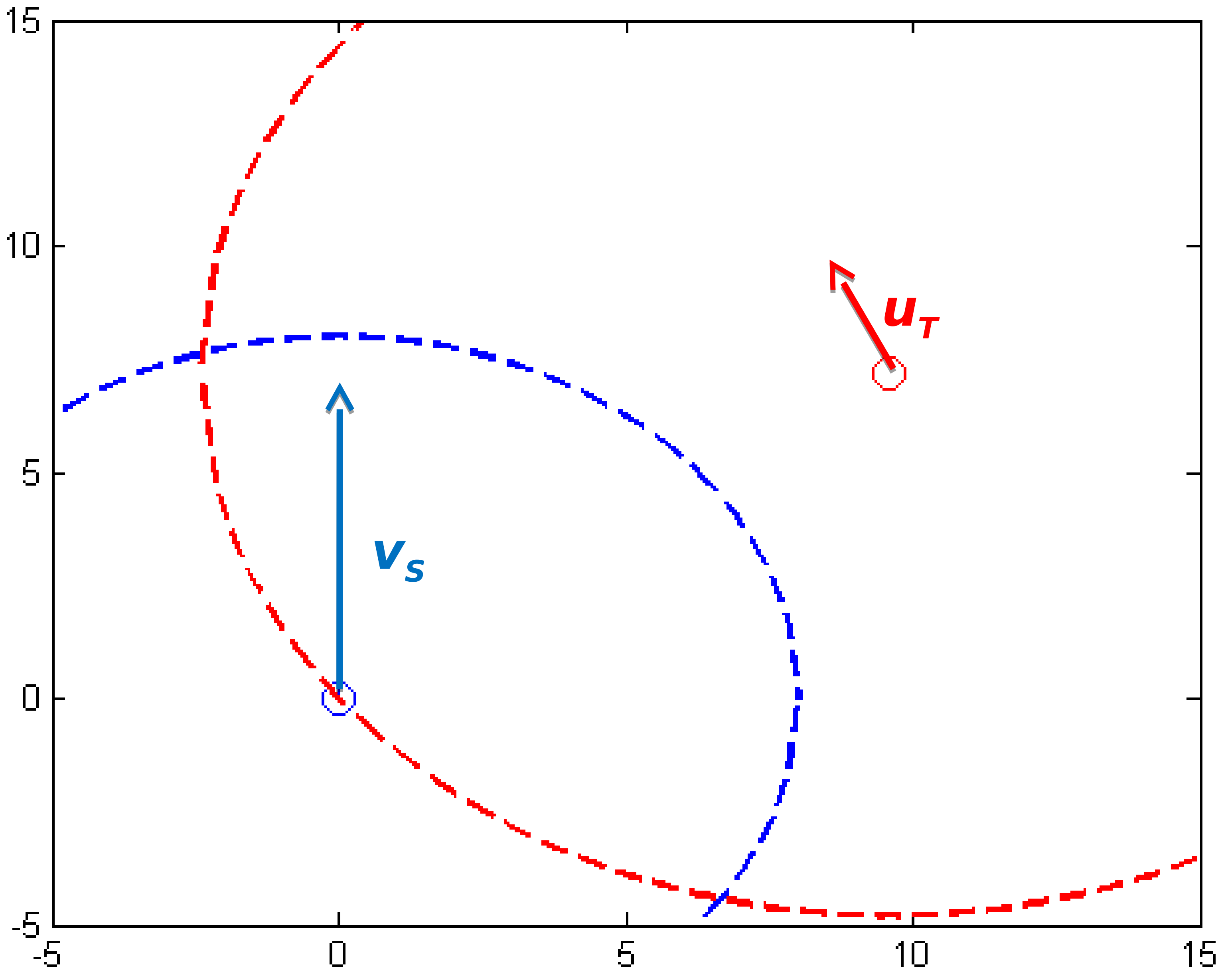
- With counter-detection the target can now decrease its lateral range
  - Move closer to approach searcher

- Thus there will be situations where a target without counter-measures will be not be detected . . .
  - . . . But a target with counter-measures can be detected

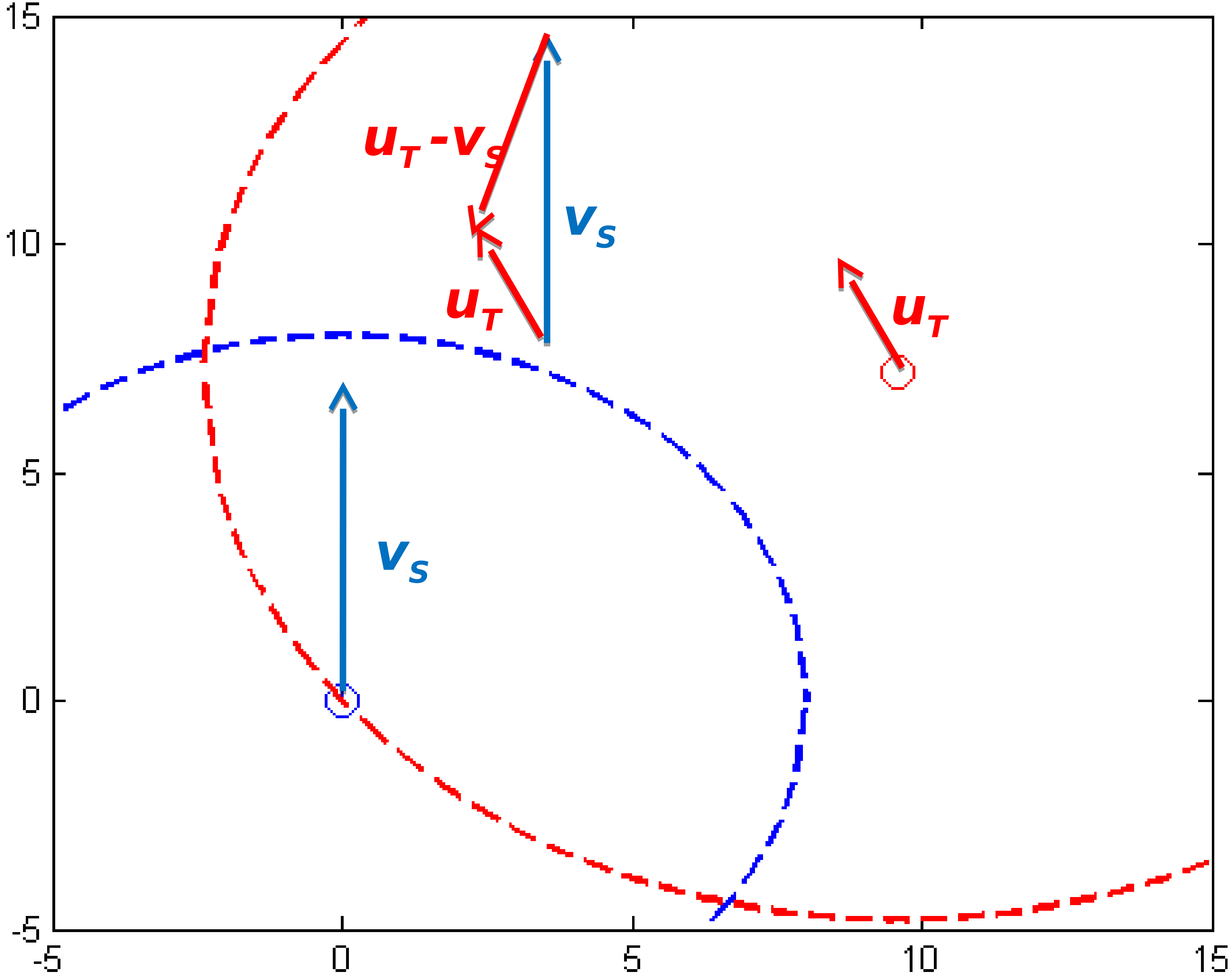
- Impact: increases our effective sweepwidth
  - Greater than $2*R_s$

- *d*: maximum distance off searcher track at counter-detection, such that target can still close to within searcher range R_s

Target Motion Relative to Searcher



Target Motion Relative to Searcher



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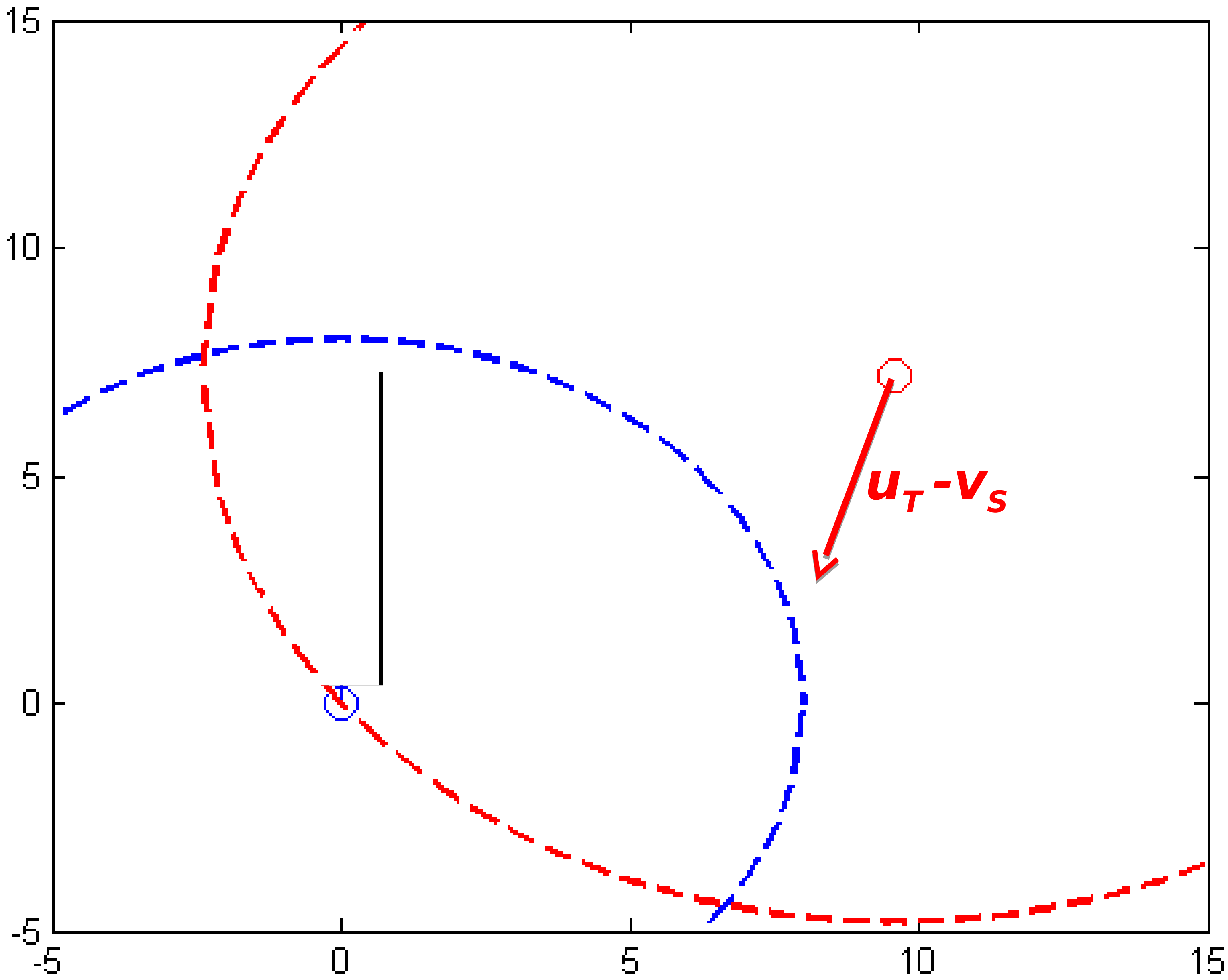
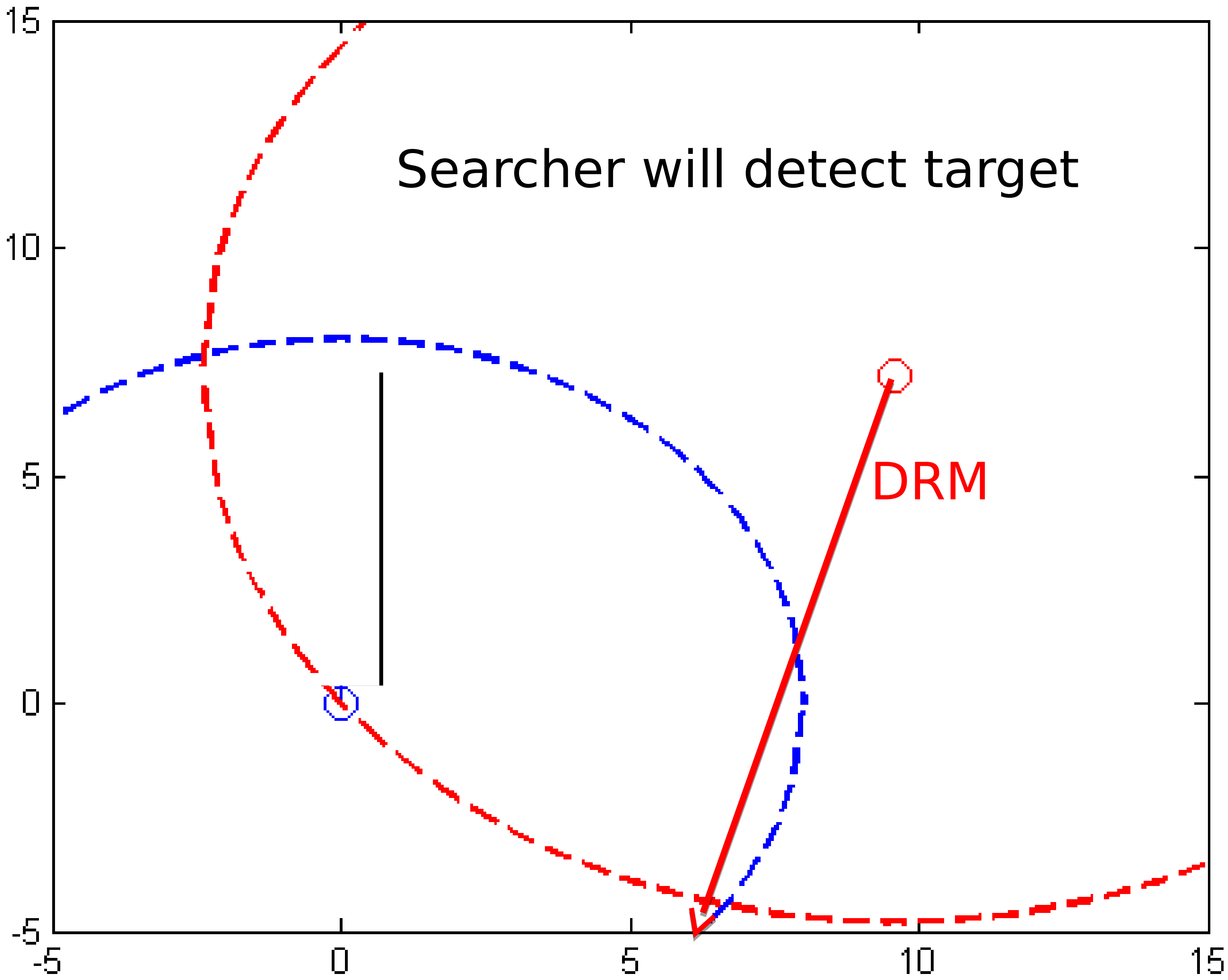


Target Motion Relative to Searcher



Target's Approach Heading

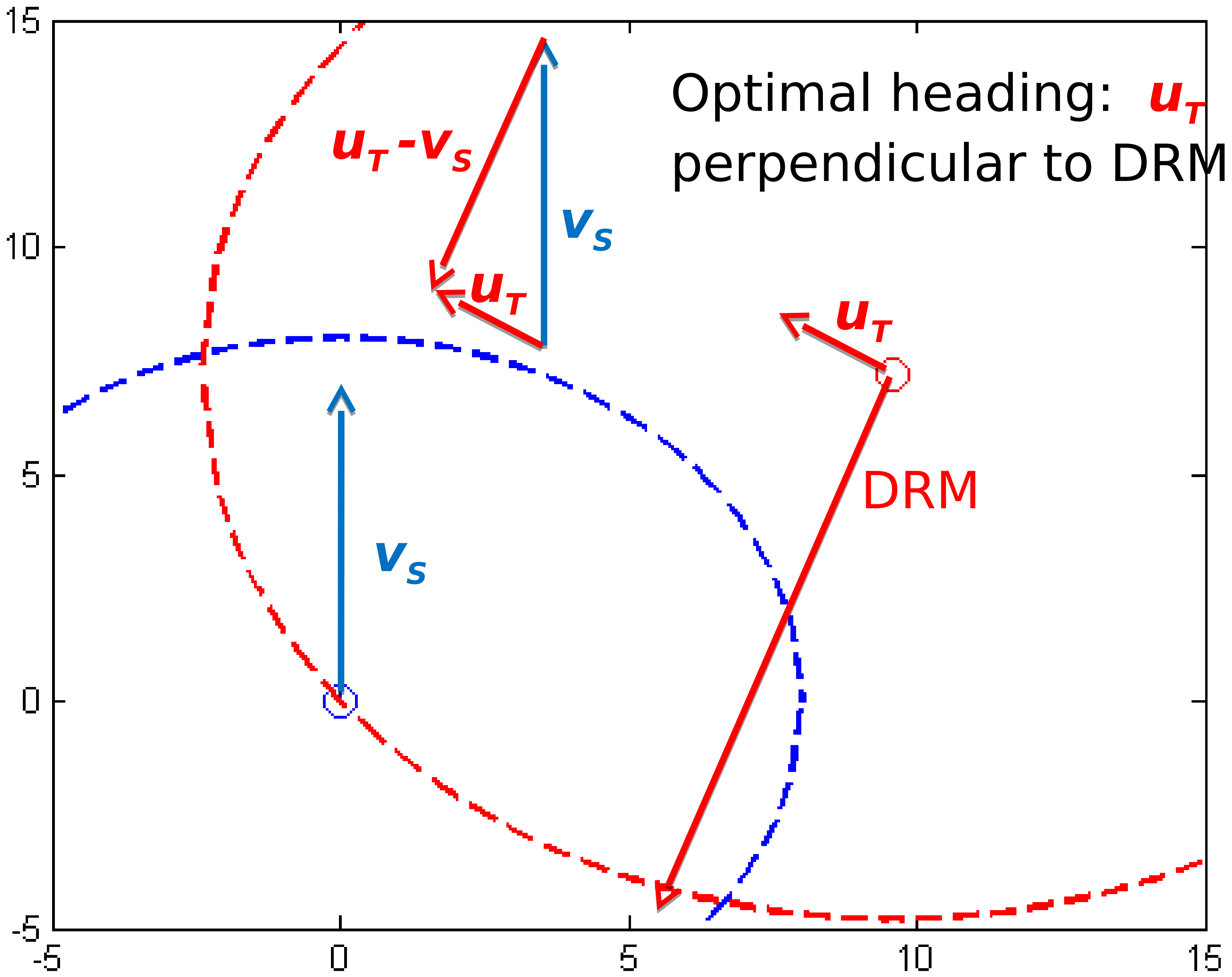
$$w_e = 2d$$

- How do we find *d* ?

- As with the evasion situation, we want the target's velocity vector perpendicular to the DRM

- Target chooses velocity vector $\boldsymbol{u_T}$ such that $\boldsymbol{u_T}$ is heading toward the searcher and the vectors $\boldsymbol{u_T}$ and $\boldsymbol{u_T - v_S}$ are perpendicular

Target's Approach Heading



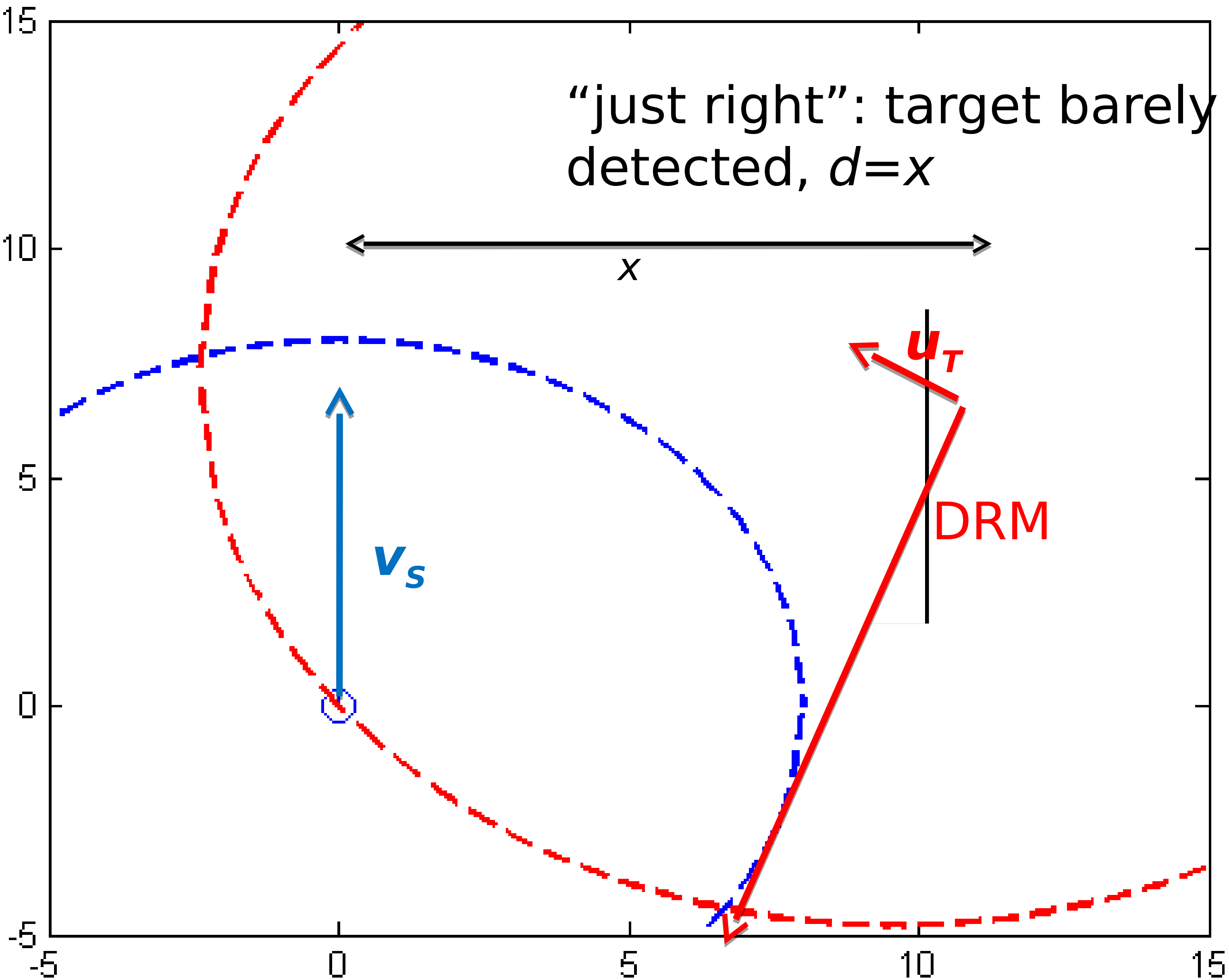
Effective Sweepwidth: Approach

$$w_e = 2d$$

- How do we find *d* ?

- As with the evasion situation, we can find *d* by finding the point where the DRM (given optimal perpendicular heading) is tangent to the searcher's sensor footprint

Determination of *d*



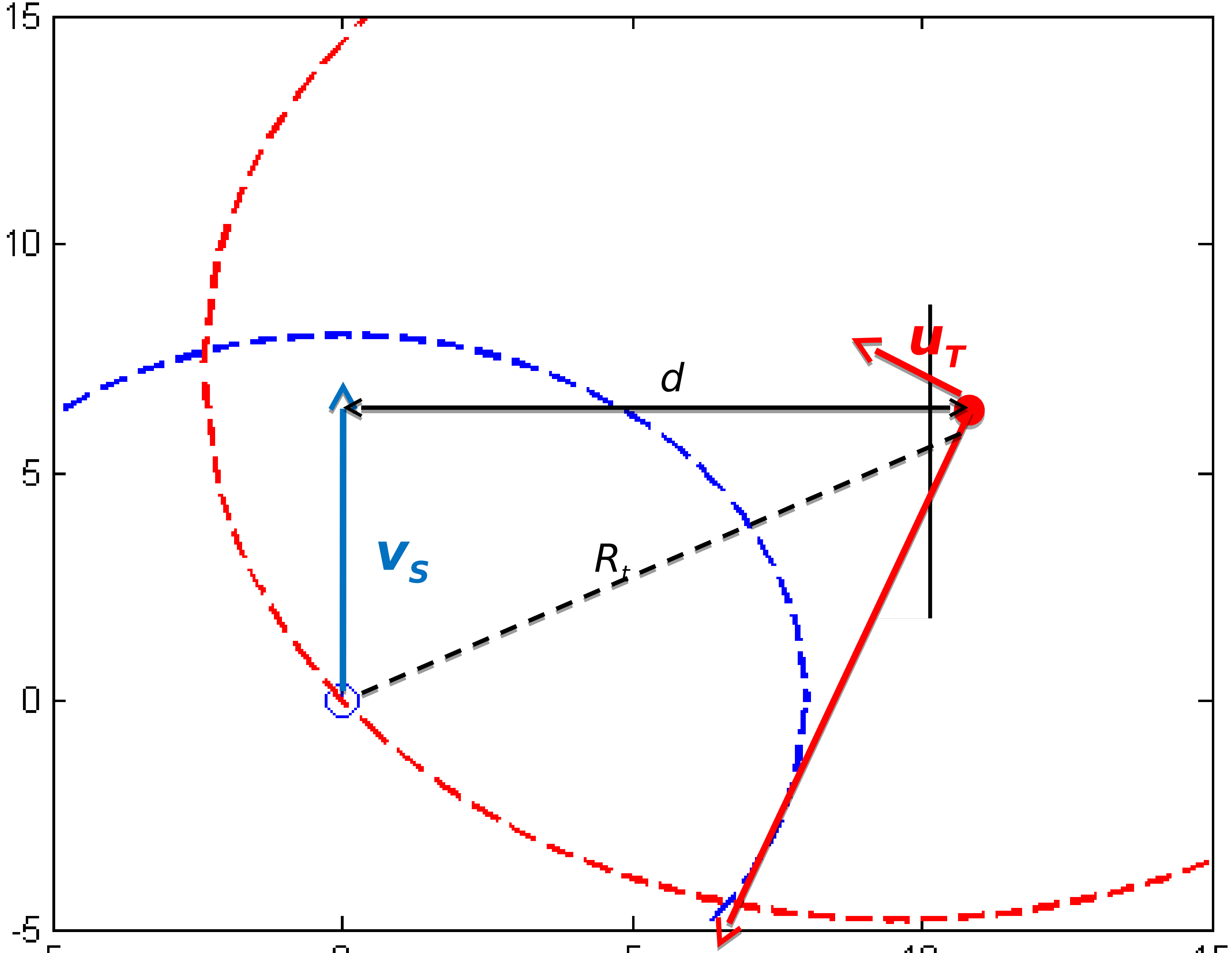
Effective Sweepwidth: Approach

$$w_e = 2d$$

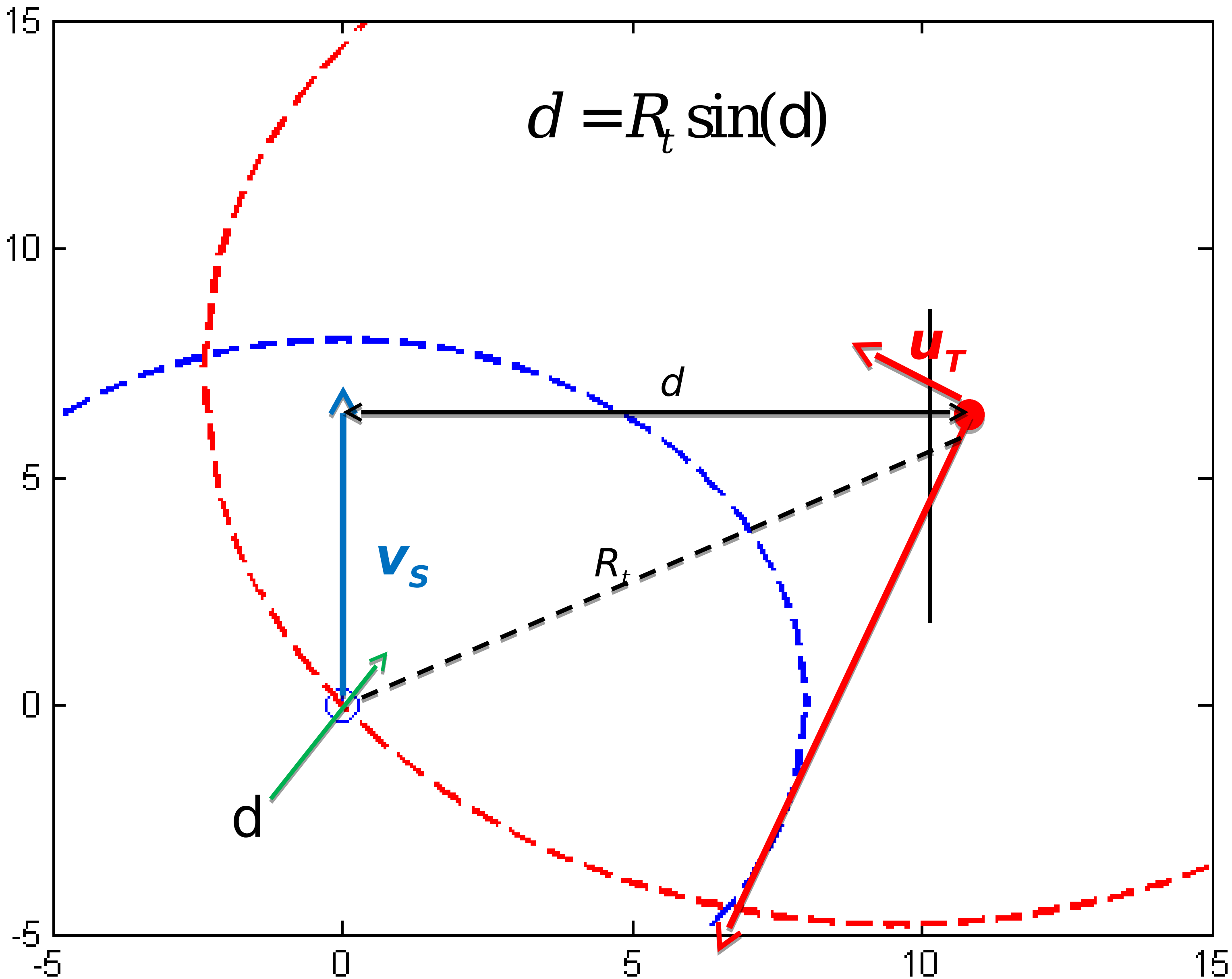
- How do we find *d* ?

- Can go through a similar geometric derivation to find *d* by comparing triangles and angles

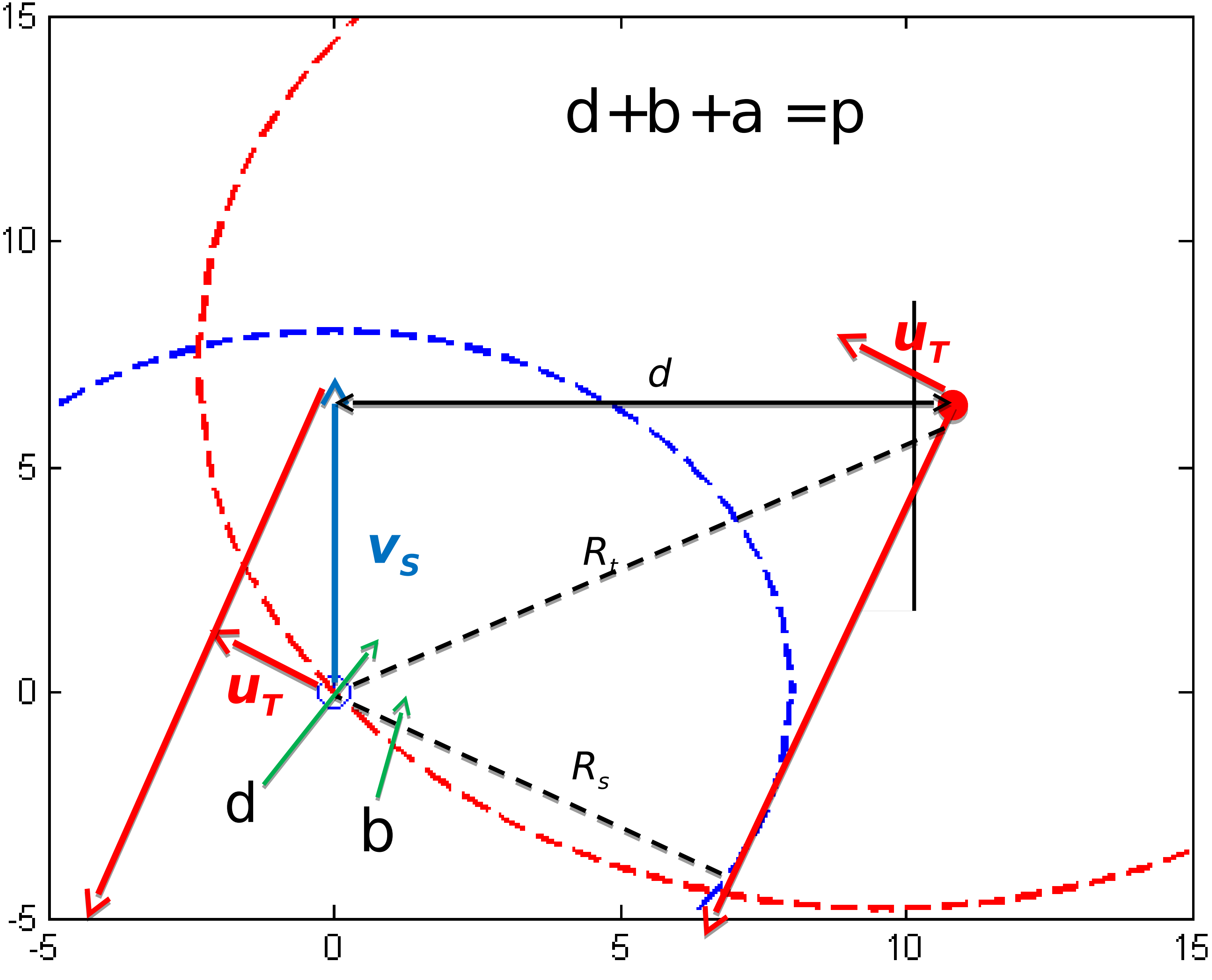
Determination of *d*



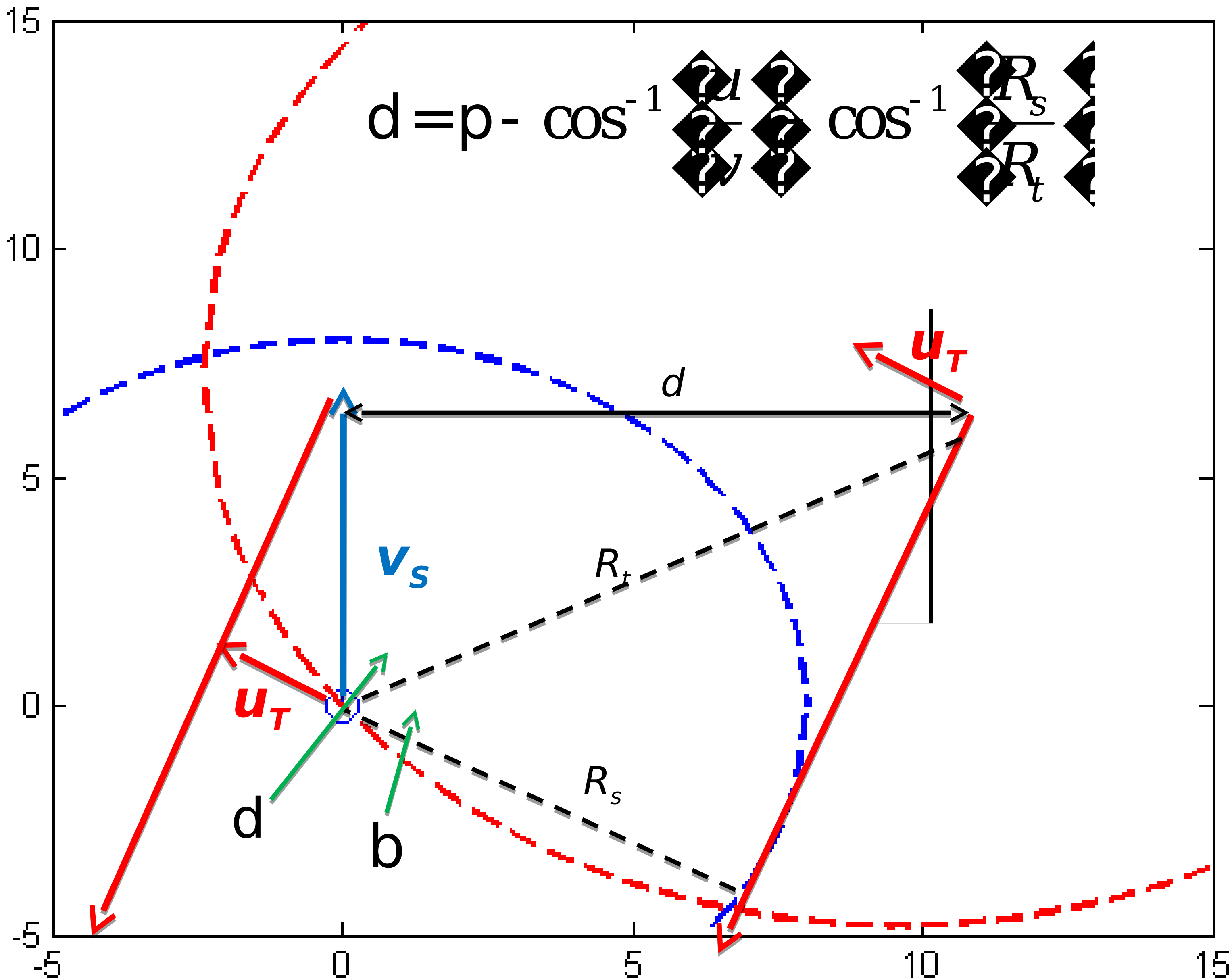
Determination of *d*



Determination of *d*



Determination of *d*



Approach Summary

$$w_e = 2d \qquad d = R_t \sin(\mathrm{d}) \qquad \mathrm{d} = \mathrm{p} - \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)$$

$$\boxed{w_e = 2R_t \sin\left(\mathrm{p} - \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right)}$$

- Only valid for
  - v > u : otherwise eventually target reaches searcher footprint and $w_e = 2R_t$
  - $R_t > R_s$: otherwise counter-detections worthless and $w_e = 2R_s$
- Also only valid for $a^2 + b^2 > 1 \Rightarrow \frac{u}{v} < \sqrt{1 - \left(\frac{R_s}{R_t}\right)^2}$
- If $\frac{u}{v} > \sqrt{1 - \left(\frac{R_s}{R_t}\right)^2}$ target has sufficient speed to always reach searcher footprint $w_e = 2R_t$

Numerical Example

- $R_s = 8$
- $R_t = 12$
- $v = 9$
- u = 3

- No countermeasures: sweepwidth w= $2R_s$ = 16nm
- With counter-detection

$$w_e = 2R_t \sin\left(\pi - \cos^{-1}\left(\frac{u}{v}\right) - \cos^{-1}\left(\frac{R_s}{R_t}\right)\right) = 21.05\text{nm}$$

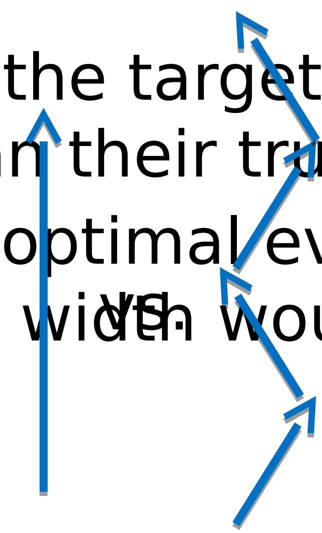
- If $\frac{u}{v} > \sqrt{1-\left(\frac{R_s}{R_t}\right)^2}$ then $w_e = 2R_t$
  - E.g., target increase speed $\frac{u}{v} > \sqrt{1-\left(\frac{R_s}{R_t}\right)^2} \Rightarrow u > 6.71$

Counter Counter-measures

- Can Searcher do anything to thwart counter-detection?
  - Or at least reduce the effectiveness of counter-detection

- We assume searcher and target know parameters: u, v, R_s, R_t
  - And searcher heading

- If searcher took actions to make the target believe v and R_s were something other than their true values, then the target would take a suboptimal evasion heading and the effective sweep width would increase (evasion case)



Other Considerations

- Sensor ranges may depend upon velocities

$$R_s(u,v), \quad R_t(u,v)$$

- Which velocity should the searcher and target choose (subject to constraints)?

- Evasion situations lead to different objectives
  - Searcher wants to maximize effective sweepwidth
  - Target wants to minimize effective sweepwidth

- Shooter situations lead to different objectives
  - Searcher wants to minimize effective sweepwidth
  - Target wants to maximize effective sweepwidth

- Rescue situations both want to maximize effective sweepwidth